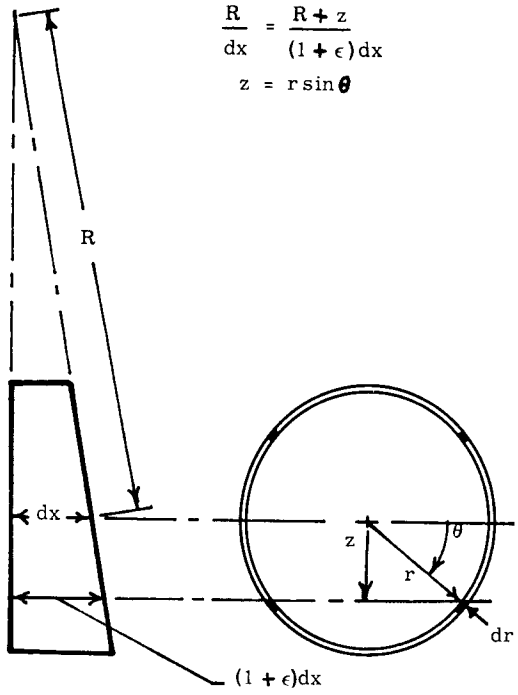

FIG. 1. Notation for a circular cross section

For small deflections, the radius of curvature is related to the deflection of the neutral axis, y, and the axial dimension, x, by $1/R = d^2y/dx^2$. Isochronous creep test results or tensile test results interrelating stresses, σ, and strains, ϵ, can be described by a power function of the following form:

$$\epsilon = A(T)\sigma^n \quad \text{or} \quad \sigma = (\epsilon/A)^{1/n} \tag{2}$$

where $A(T)$ is a function of the temperatures. Since the temperature, T, is a known function of the radius, A can be written as a function of the radius, r.

For a tube of thickness dr and radius r, the moment contribution for the curvature R is

$$dM = 4\int_0^{\pi/2} \sigma(r \sin\theta) r d\theta\, dr \tag{3}$$

Substituting for σ from Eq. (2), for ϵ from Eq. (1), and introducing the approximation for the radius of curvature yields the incremental moment equation. The total moment is obtained by integrating the incremental moment equation with respect to the radius, r. The resulting moment-deflection relation is

$$M(x) = \int_{R_i}^{R_0} dM = B(y'')^{1/n}, \tag{4}$$

where R_i is the inner tube radius ($R_i = 0$ for a rod); R_0, the outer tube radius; and

$$B = 2\sqrt{\pi}\,\frac{\Gamma(1 + 1/2n)}{\Gamma[1 + (n+1)/2n]}\int_{R_i}^{R_0}\left(\frac{r}{A}\right)^{1/n} r^2\, dr.$$

The function B is independent of the bending conditions within the rod and is a function of the power dependence of the stress-strain relations and the radial dependence of the material parameter, A. Thus, for any particular problem B is a constant and Eq. (4) can be integrated to obtain the deflections.

A simply supported beam of length L with a uniform load w has the following moment distribution, M:

$$M = (w/2)\,(Lx - x^2) \tag{5}$$

where x = distance from a support. Solving Eq. (4) for the second derivative of the deflection

$$y'' = (M/B)^n.$$

Substituting in the moment distribution (5), integrating, and evaluating boundary conditions yields:

$$|y|_{\max} = \left(\frac{w}{2B}\right)^n \int_0^{L/2} dx \int_x^{L/2} (Lt - t^2)^n\, dt$$

$$= \left(\frac{w}{2B}\right)^n \int_0^{L/2} x(Lx - x^2)^n\, dx.$$

When n is an integer

$$|y|_{\max} = \left(\frac{w}{B}\right)^n \left(\frac{L}{2}\right)^{2n+2}\left[\frac{1}{(2n+1)(2n-1)\cdots 3} - \frac{1}{2^{n+1}(n+1)}\right].$$

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Note on the Thermal Neutron Spectrum in a Diffusing Medium[1]

A paper by Hurwitz and Nelkin (*1*) considers the energy-dependent thermal diffusion equation in a region free of external sources. Hurwitz and Nelkin consider two similar cases:

(a) The steady-state diffusion of neutrons from a thermal plane source in an infinite medium and

(b) The time-dependence of the thermal flux following a pulse of fast neutrons.

The present authors have misgivings concerning the basic assumption of flux separability made in the Hurwitz and Nelkin paper which they feel may not be correct. In case (a), it is assumed [see Eq. (9) of ref. *1*] that $\phi(\mathbf{r}, E) = \Omega_\mu(\mathbf{r})\cdot\phi_\mu(E)$. In case (b) [see Eq. (13) of ref. *1*], the assumed $\phi(E, \mathbf{r}, t) = \phi_\lambda(E)\Omega_B(\mathbf{r})\cdot e^{-\lambda t}$ where λ is explicitly taken to be independent of energy.[2] We wish to make the following comments:

Case (a). Consider a strong absorbing medium in which

[1] This communication has been presented by one of the authors (G. de Coulon) to the faculty of the University of Michigan, in partial fulfillment of the requirements for the degree of Master of Science.

[2] This treatment is also followed in a later paper by M. Nelkin (*J. Nuclear Energy* **8**, 48 (1958)).

the absorption cross section obeys the $1/v$ law. If the scattering mean free path (taken to be energy-independent) is large, the source flux will be attenuated primarily by absorptions. Due to the preferential absorption of low energy neutrons, the spectrum will become increasingly harder as the distance from the source increases, and no equilibrium situation will be reached.

It might be argued that far enough from the source the spectrum will become so hard that $\bar{\Sigma}_a$ will become small, and asymptotic equilibrium will be attained. This raises three questions, assuming the above (unproven) argument is accepted:

(1) How far from the source will this equilibrium be reached?

(2) What relation, if any, will the equilibrium spectrum have to the spectrum in a reactor with the same value of $\Sigma_a/\xi\Sigma_s$?

(3) True equilibrium (i.e., separability) is clearly reached only in the limit $\Sigma_a \to 0$, which is a trivial case since the flux is constant in position. It remains to be shown that this is a stable equilibrium, i.e., that for small Σ_a there will be an exponential decay characterized by a relaxation length given by Eq. (12) in ref. *1*.

Even if questions (1) and (3) are answered satisfactorily by the quasi-rigorous physical arguments frequently given, question (2) is still unanswered, and of vital importance if the diffusion length measured by relaxation of a thermal source is to be used in reactor calculations.

Case (b). Consider a medium of buckling B^2, containing a material whose transport cross section is energy-independent, and whose absorption cross section is small and obeys the $1/v$ law. Then, if the material has an infinite mass, the flux at any time t will be related to the initial flux by

$$\phi(E, t) = \phi(E, 0) \exp [-v(\Sigma_a + DB^2)t] \qquad (1)$$

and is clearly inseparable in energy and time. If we now allow the mass to become finite, but still large, the contention is that energy exchange will eventually bring about an equilibrium situation. Again, this raises questions, assuming this argument is accepted:

(1) How long after the initial burst will equilibrium be reached?

(2) What relation will there be between the equilibrium spectrum and the spectrum in a reactor?

The problems raised with respect to case (a) could easily be answered experimentally by measuring the neutron spectrum as a function of distance from the source in a large moderating block, perhaps by means of chopper experiments. Case (b) is being investigated theoretically by S. Purohit at the University of Michigan, who is solving the time-dependent equations without assuming separability.

It seems reasonable that the questions raised here should be answered in some rigorous fashion before the parameter measured either by relaxation or pulsed techniques are used in reactor design.[3] If separability cannot be established for case (b), this fact might well be at least partially responsible for the so-called "diffusion-cooling" effect discussed in ref. *1*. Even if separability is established within reasonable time, it might well be that the equilibrium spectrum differs so drastically from the quasi-Maxwellian equilibrium in a reactor that perturbation or variational techniques for obtaining the eigenvalues are not sufficiently accurate. Note that for $\Sigma_a \gg DB^2$ separability seems a priori much more likely since $v\Sigma_a$ is constant; see Eq. (1). However, this condition is not fulfilled in pulsed experiments in graphite, beryllium, or D_2O.

It is a pleasure to acknowledge the valuable comments received on an earlier version of this letter from H. Hurwitz, M. Nelkin, B. E. Simmons, E. R. Cohen, and G. von Dardel.

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1. H. Hurwitz and M. Nelkin, *Nuclear Sci. and Eng.* **3,** 1–10 (1958).

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[3] It should also be noted that the problem of "mode-mixing" due to variation of extrapolated boundary with energy contributes a great deal of uncertainty. This is a separate problem, however, from those discussed here.