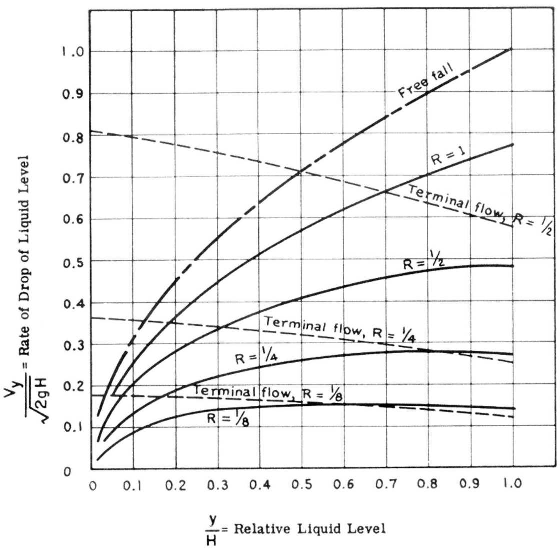

FIG. 2. Variation of free surface velocity with liquid level in tank. $L_1 = L_2 = H$; $n = 0$; $F = 0$.

for the selected pressurization, an area ratio of 1:4 produced a rate of drop in liquid level approximating free fall. The terminal flow approximation is in this case rather poor even for small area ratios.

The effect of friction on the system was observed by arbitrarily setting $F = 3$, which represented a friction loss of three drain pipe velocity heads. It was noted that the velocities for corresponding area ratios were smaller and that terminal flow was established sooner. Although terminal flow was a better approximation to true flow, it was not a satisfactory approximation, for moderate area ratios, of the early stage flow.

The effect of drain pipe length on the development of flow was observed by considering a configuration having a small area ratio typical of a water tank with drain pipe attachment. An area ratio of 0.01 was selected, and the velocity-displacement relationship plotted for lengths of drain pipe of zero, H, and $5H$. The curves indicated, in common, that the inertia transient was completed and terminal flow started when the maximum rate of flow had been more or less attained.

In examining the development of flow when a very long drain pipe, equal to $5H$, is attached to the tank, it is found that the peak velocity is not attained until about half the tank has been drained. This indicates that a long drain pipe produces a long inertia transient, and that computations of discharge based on the prevailing head lead to incorrect results—even when the area ratio is small. When the drain pipe is of moderate length, equal to H, then the peak velocity is reached when less than one-thousandth of the volume of liquid has been drained. In this case, it can be assumed, for practical purposes, that terminal flow is established immediately.

It is fairly apparent that to obtain the rapid drainage required for scramming a reactor it is necessary to have a large discharge area. Rather than use a very large pipe or group of pipes, it is expedient to use an annular weir discharge. In this arrangement the length of the drain pipe $L = L_1 + L_2$ is essentially zero. The solution of Eq. (2), disregarding pressurization and friction, is

$$\frac{V_y^2}{2gH} = \frac{R^2}{1 - 2R^2}\left[1 - \frac{y}{H} - \left(1 - \frac{y}{H}\right)^{(1-R^2)/R^2}\right]. \quad (5)$$

For the ratio $R = 1/\sqrt{2}$, Eq. (5) becomes

$$\frac{V_y^2}{2gH} = \left(1 - \frac{y}{H}\right)\ln\left(\frac{1}{1 - (y/H)}\right). \quad (6)$$

As $1/\sqrt{2}$ is a very large area ratio, it is well represented in the early stage flow by free fall. An interesting feature of the flow, with no drain pipe length, is that the initial acceleration at which the level in the tank falls is always g regardless of the height of liquid in the tank or area of discharge.

The time required for the complete draining of the tank when $R = 1/\sqrt{2}$ is obtained bo integrating Eq. (6). The definite integral is

$$T = (H/\sqrt{2gH})\int_0^1 z^{-1/2}[-\ln z]^{1/2}\,dz = \left(\frac{H}{2g}\right)^{1/2}\frac{\Gamma(1/2)}{(1/2)^{1/2}} = \left(\frac{\pi H}{g}\right)^{1/2} \quad (7)$$

where $z = 1 - (y/H)$. It is found from the above equation that until about three-tenths of the tank has been drained the time-displacement relationship is practically that of free fall. The divergence becomes more significant as the draining of the tank nears completion. To complete the draining, 25% more time is required than the time it takes for a body to fall freely through this distance. Terminal flow in this case is not at all representative of the true state of affairs.

REFERENCE

1. D. BURGREEN, "Hydrodynamics of a Liquid Poison Scram System," *Nuclear Sci. and Eng.* **4**, 82 (1958).

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Plastic Bending of Rods or Tubes with Radial Temperature Distributions

Horizontally loaded reactor fuel elements are subject to bending stresses which may limit the distance between fuel element supports. Since these fuel elements have radial temperature distributions, standard (*1*) methods of plastic analysis cannot be used to determine the deflections. This note contains the equations necessary to extend existing methods of plastic bending analysis to cases of rods or tubes with radial temperature distributions.

Assuming plane sections perpendicular to the neutral axis remain plane after bending, the equation relating the axial strain, ϵ, and the curvature, R (Fig. 1), is

$$\epsilon = \frac{z}{R} = \frac{r \sin\theta}{R}$$

where r is the distance from the center; z, the distance from the neutral axis; and θ, the angle between r and neutral axis.

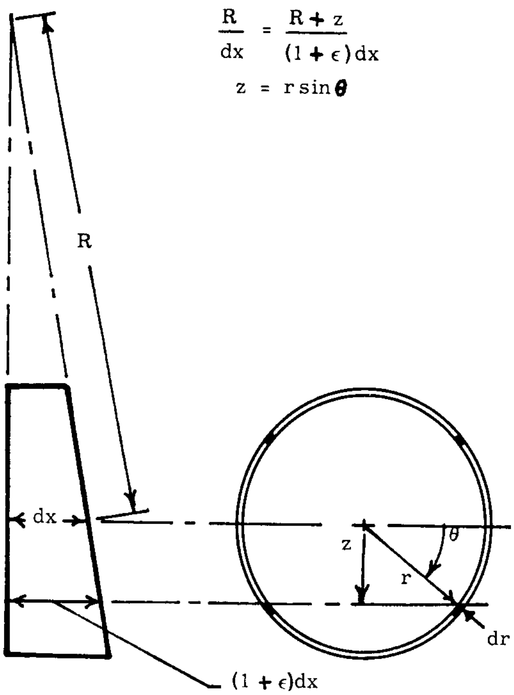

FIG. 1. Notation for a circular cross section

For small deflections, the radius of curvature is related to the deflection of the neutral axis, y, and the axial dimension, x, by $1/R = d^2y/dx^2$. Isochronous creep test results or tensile test results interrelating stresses, σ, and strains, ϵ, can be described by a power function of the following form:

$$\epsilon = A(T)\sigma^n \quad \text{or} \quad \sigma = (\epsilon/A)^{1/n} \tag{2}$$

where $A(T)$ is a function of the temperatures. Since the temperature, T, is a known function of the radius, A can be written as a function of the radius, r.

For a tube of thickness dr and radius r, the moment contribution for the curvature R is

$$dM = 4 \int_0^{\pi/2} \sigma(r \sin \theta) r d\,\theta\, dr \tag{3}$$

Substituting for σ from Eq. (2), for ϵ from Eq. (1), and introducing the approximation for the radius of curvature yields the incremental moment equation. The total moment is obtained by integrating the incremental moment equation with respect to the radius, r. The resulting moment-deflection relation is

$$M(x) = \int_{R_i}^{R_0} dM = B(y'')^{1/n}, \tag{4}$$

where R_i is the inner tube radius ($R_i = 0$ for a rod); R_0, the outer tube radius; and

$$B = 2\sqrt{\pi}\, \frac{\Gamma(1 + 1/2n)}{\Gamma[1 + (n+1)/2n]} \int_{R_i}^{R_0} \left(\frac{r}{A}\right)^{1/n} r^2\, dr.$$

The function B is independent of the bending conditions within the rod and is a function of the power dependence of the stress-strain relations and the radial dependence of the material parameter, A. Thus, for any particular problem B is a constant and Eq. (4) can be integrated to obtain the deflections.

A simply supported beam of length L with a uniform load w has the following moment distribution, M:

$$M = (w/2)\,(Lx - x^2) \tag{5}$$

where x = distance from a support. Solving Eq. (4) for the second derivative of the deflection

$$y'' = (M/B)^n.$$

Substituting in the moment distribution (5), integrating, and evaluating boundary conditions yields:

$$|y|_{\max} = \left(\frac{w}{2B}\right)^n \int_0^{L/2} dx \int_x^{L/2} (Lt - t^2)^n\, dt$$
$$= \left(\frac{w}{2B}\right)^n \int_0^{L/2} x(Lx - x^2)^n\, dx.$$

When n is an integer

$$|y|_{\max} = \left(\frac{w}{B}\right)^n \left(\frac{L}{2}\right)^{2n+2} \left[\frac{1}{(2n+1)(2n-1)\cdots 3} - \frac{1}{2^{n+1}(n+1)}\right].$$

REFERENCE

1. A. PHILLIPS, "Introduction to Plasticity." Ronald Press, New York, 1956.

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Note on the Thermal Neutron Spectrum in a Diffusing Medium[1]

A paper by Hurwitz and Nelkin (*1*) considers the energy-dependent thermal diffusion equation in a region free of external sources. Hurwitz and Nelkin consider two similar cases:

(a) The steady-state diffusion of neutrons from a thermal plane source in an infinite medium and

(b) The time-dependence of the thermal flux following a pulse of fast neutrons.

The present authors have misgivings concerning the basic assumption of flux separability made in the Hurwitz and Nelkin paper which they feel may not be correct. In case (a), it is assumed [see Eq. (9) of ref. *1*] that $\phi(\mathbf{r}, E) = \Omega_\mu(\mathbf{r}) \cdot \phi_\mu(E)$. In case (b) [see Eq. (13) of ref. *1*], the assumed $\phi(E, \mathbf{r}, t) = \phi_\lambda(E)\Omega_B(\mathbf{r}) \cdot e^{-\lambda t}$ where λ is explicitly taken to be independent of energy.[2] We wish to make the following comments:

Case (*a*). Consider a strong absorbing medium in which

[1] This communication has been presented by one of the authors (G. de Coulon) to the faculty of the University of Michigan, in partial fulfillment of the requirements for the degree of Master of Science.

[2] This treatment is also followed in a later paper by M. Nelkin (*J. Nuclear Energy* **8**, 48 (1958)).