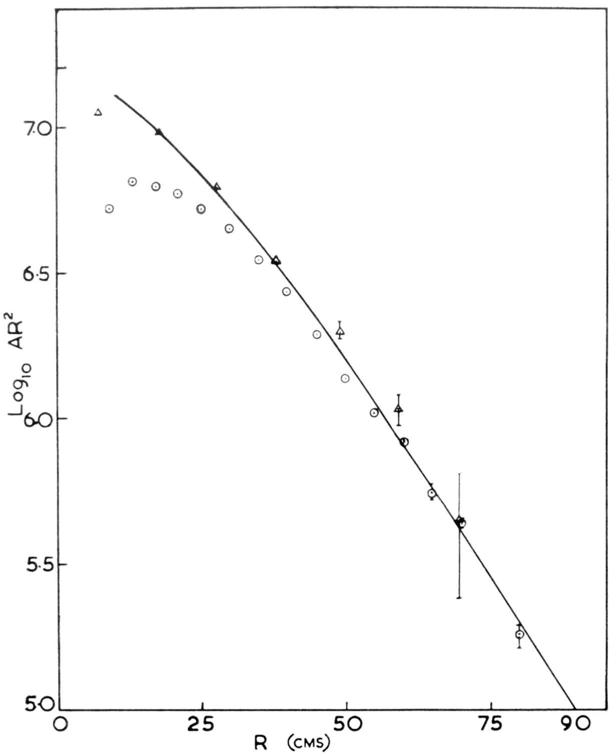

FIG. 3. Log AR^2 (in arbitrary units) as a function of distance R (in cm) in pure water. △ Phosphorous detector (above 1 Mev). ○ Indium detector (1.46 ev). — Theoretical curve for phosphorous detector using the Goldstein *et al.* calculations.

and 3 mm thick. Indium detectors, used to detect resonance neutron flux (1.46 ev), were of the same cross-section area and 0.12 mm thick.

The saturation activity A of the foils was measured as a function of distance R taken from the center of the target tube. Figure 2 shows curves of log AR^2 (relative values) as a function of distance R obtained with In and P detectors in 6-in. Pb + H_2O medium. It is seen that more than 60 cm from the source, the rate of attenuation of the two curves is about the same. This is in agreement with the theoretical predictions (*3*) that at large distances from the source, flux at all energies will fall off, apart from a geometrical factor, as $\exp(-N\sigma_{tr}R)$, where $N\sigma_{tr}$ is the macroscopic transport cross section and E is the source energy. Assuming the neutron flux to fall off as $[\exp(-R/L)]/R^2$, the value of relaxation length L between 60 and 70 cm was found to be 13.6 ± 0.5 cm. Figure 3 shows similar curves in the case of pure water. As in Fig. 2, at large distances the rate of attenuation of P and In curves is about the same. The solid curve in Fig. 3 represents the expected theoretical response of a P detector based on the results of Goldstein *et al.* (*4*). In their treatment, using the moments method to solve the Boltzmann equation, the energy spectrum of neutrons as a function of distance has been obtained from a 14.03 Mev source in water. The cross-section curve for the $P^{31}(n,p)Si^{31}$ reaction (*5*) as a function of energy was integrated over the theoretical curve at different distances. The curve thus obtained represents the expected theoretical response of a P detector from a 14.03 Mev source in water. The experimental points have been normalized with the theoretical curve at 17.5 cm. It is seen that experimental points higher than 45 cm lie above the theoretical curve. This has also been observed in the dose measurements by Caswell *et al.* (*1*). The accuracy of the theoretical calculations is estimated by the authors as 15%. There is an inaccuracy of about 8% in the determination of $P^{31}(n,p)Si^{31}$ cross section and the values between 10 Mev and 14 Mev have not been measured. For the present calculation the cross-section curve was joined smoothly between 10 Mev and 14 Mev. It was also noticed that a variation of 10% in the cross section in this region caused a negligible change in the slope of the theoretical curve. Between 60 and 70 cm, the relaxation length L of the theoretical curve was found to be 15 cm. An uncertainty of about 2 cm in the determination of the exact center of the target was present in all the measurements. No correction has been made for the change in flux caused by the presence of the aluminum tube (3.8 cm diameter) used to bring in the deuteron beam.

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V. P. DUGGAL
S. M. PURI
K. SRI RAM

Atomic Energy Establishment
Trombay, India
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The Inertia Transient in Reactor Draining

In analyzing the problem of reactor draining, or of tank draining in general, three types of transients may be taken into consideration. The most rapid of these transients, which includes inertia and compressibility effects, is the sonic transient. An intermediate transient is obtained by disregarding compressibility and taking into account only the inertia of the fluid, and is termed here the inertia transient. The slowest of the transients results from disregarding both compressibility and inertia effects and assuming that the rate of discharge is governed only by the prevailing head. It is designated as terminal flow, since it characterizes the terminal phase of the inertia transient.

The order of magnitude of time involved in reactor draining problems, arising from reactor control or reactor safety studies, requires that cognizance be taken of the inertia transient. The results presented here have application in the design of reactor control, or reactor scramming systems

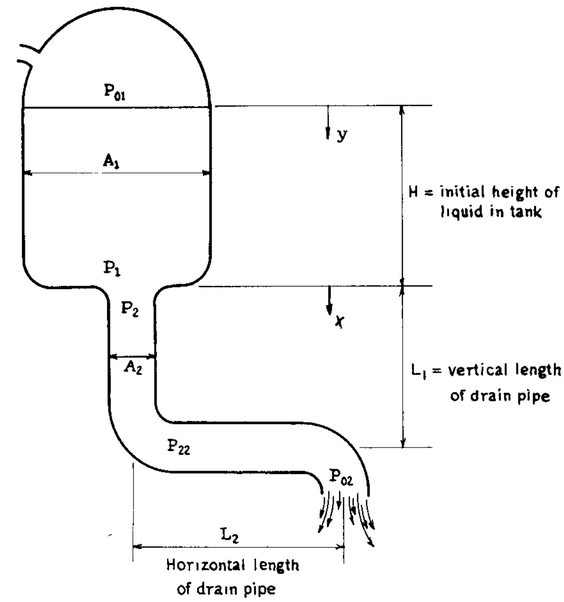

FIG. 1. Tank and drain pipe arrangement

which depend upon the draining of moderator or reflector liquid, and they may be of use in the evaluation of reactor hazards resulting from the loss of coolant or other reactor liquid.

A general study of the inertia transient in a conservative mass and energy system is discussed elsewhere (*1*). The present system is nonconservative.

The system consists of a large pressurized tank from which liquid drains through a pipe having a vertical and a horizontal length. It drains under the influence of gravity, assisted in some cases by a gas pressure on the surface of the liquid. Figure 1 shows the arrangement of tank and drain pipe. Proceeding in a manner similar to that used in reference *1*, the summation of forces on the slugs of liquid in the tank and pipes, together with the energy and continuity relationships, at the junction of pipe and tank results in the equation:

$$\left[L + \frac{A_2}{A_1}H - \left(\frac{A_2}{A_1}\right)^2 x\right]\frac{d^2x}{dt^2} + \frac{1}{2}\left[1 + F - \left(\frac{A_2}{A_1}\right)^2\right]\left(\frac{dx}{dt}\right)^2 - g\left[H + \frac{P_{01}}{w} + L_1 - \frac{A_2}{A_1}x\right] = 0. \tag{1}$$

It is expressed in terms of x, the displacement of a particle in the drain tube, but could be expressed just as readily in terms of the drop of level in the tank, y, by use of the continuity equation $A_1 y = A_2 x$. In deriving the above equation, P_{02}, the pressure at the end of the drain pipe has been assumed to be atmospheric (zero gage). In Eq. (1), $L = L_1 + L_2$ and F represent the number of velocity heads lost through pipe friction and would include entrance, exit, bend, and valve losses in addition to the straight pipe losses.

Equation (1) is a nonlinear differential equation with variable coefficients. The first integral, in terms of the tank level y, is obtainable, however. It gives the free surface velocity as

$$V_y = \frac{dy}{dt} = \left\{\frac{2gR^2}{1 + F - R^2}\left[(n + 1)H + L_1 + \frac{R(L + RH)}{1 + F - 2R^2}\right]\left[1 - \left(1 - \frac{Ry}{L + RH}\right)^{(1+F-R^2)/R^2}\right] - \frac{2gR^2y}{1 + F - 2R^2}\right\}^{1/2}. \tag{2}$$

In the above $R = A_2/A_1$ and $nH = P_{01}/w$.

The solution of Eq. (1) with the inertia term $d^2x/dt^2 = 0$ is the solution of the well-known tank draining problem in which the discharge is governed by the momentary head and the inertia of the fluid to a change in velocity is disregarded. It is a close approximation of the flow for the case when both the area ratio is small and the length of drain pipe is small. Under zero inertia conditions, the solution to Eq. (1) in terms of the y coordinate, or liquid level in the tank, is

$$V_y = \frac{dy}{dt} = R\left[\frac{2g(n + 1)H + L_1 - y}{1 + F - R^2}\right]^{1/2}. \tag{3}$$

Equation (2) has four parameters in addition to the independent variable y/H. The effect of these parameters will be examined separately. Consider first the solution when the drain pipe lengths $L_1 = L_2 = H$, when there is no pressurization, and the friction forces are neglected. Equation (2) becomes

$$\frac{V_y^2}{2gH} = \frac{R^2(2 + 2R - 3R^2)}{(1 - R^2)(1 - 2R^2)}\left[1 - \left(1 - \frac{y}{H}\cdot\frac{R}{2 + R}\right)^{(1-R^2)/R^2}\right] - \frac{y}{H}\cdot\frac{R^2}{1 - 2R^2}. \tag{4}$$

Equation (4) is plotted in Fig. 2 for several ratios of drain pipe area to tank area.

A free-fall curve, which is included for comparative purposes, shows the velocity of a free falling body, dropped from a height H. It is the special case of $R = 1$ and $L_2 = 0$. The curves in Fig. 2 show the rate of fall of the liquid surface in the tank to decrease with decreasing area ratio. The range in which the flow is governed primarily by inertia forces also becomes smaller with decreasing area ratio. This may be observed through the increasingly early blending of the true flow curves with the terminal flow curves as the area ratio becomes smaller. The dotted terminal flow curves are plotted from Eq. (3). These are seen to be fair approximations of the terminal phase of flow.

When the area ratio is quite large, as, for example, when $R = 1/2$ the velocity-displacement curve shows that the inertia forces primarily determine the character of the flow. For large area ratios the velocity-displacement curves resemble the free-fall curve and any approximation with terminal flow would not be valid. Note that the true velocity-displacement curves indicate, properly, that the velocity starts at zero and builds up, while the terminal flow curves indicate, incorrectly, that at the start of flow the velocity is $V = (2gH)^{1/2}$.

The effect of pressurization was examined by letting $n = 10$ and $L_1 = L_2 = H$. The resulting curves (not shown here) indicate that a rate of drop in liquid level, in excess of the free-fall rate, is easily obtained. It was observed that

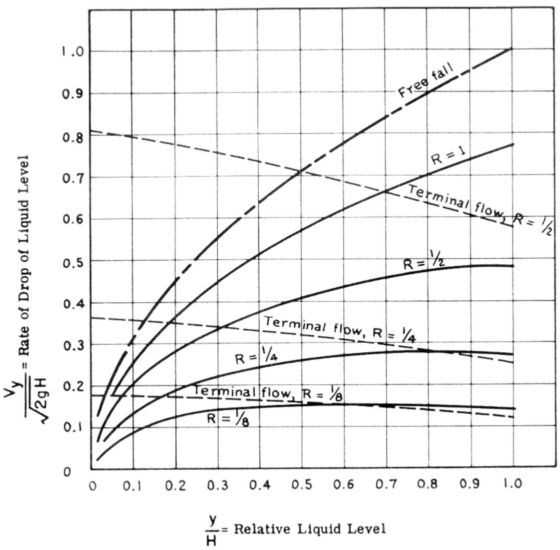

FIG. 2. Variation of free surface velocity with liquid level in tank. $L_1 = L_2 = H$; $n = 0$; $F = 0$.

for the selected pressurization, an area ratio of 1:4 produced a rate of drop in liquid level approximating free fall. The terminal flow approximation is in this case rather poor even for small area ratios.

The effect of friction on the system was observed by arbitrarily setting $F = 3$, which represented a friction loss of three drain pipe velocity heads. It was noted that the velocities for corresponding area ratios were smaller and that terminal flow was established sooner. Although terminal flow was a better approximation to true flow, it was not a satisfactory approximation, for moderate area ratios, of the early stage flow.

The effect of drain pipe length on the development of flow was observed by considering a configuration having a small area ratio typical of a water tank with drain pipe attachment. An area ratio of 0.01 was selected, and the velocity-displacement relationship plotted for lengths of drain pipe of zero, H, and $5H$. The curves indicated, in common, that the inertia transient was completed and terminal flow started when the maximum rate of flow had been more or less attained.

In examining the development of flow when a very long drain pipe, equal to $5H$, is attached to the tank, it is found that the peak velocity is not attained until about half the tank has been drained. This indicates that a long drain pipe produces a long inertia transient, and that computations of discharge based on the prevailing head lead to incorrect results—even when the area ratio is small. When the drain pipe is of moderate length, equal to H, then the peak velocity is reached when less than one-thousandth of the volume of liquid has been drained. In this case, it can be assumed, for practical purposes, that terminal flow is established immediately.

It is fairly apparent that to obtain the rapid drainage required for scramming a reactor it is necessary to have a large discharge area. Rather than use a very large pipe or group of pipes, it is expedient to use an annular weir discharge. In this arrangement the length of the drain pipe $L = L_1 + L_2$ is essentially zero. The solution of Eq. (2), disregarding pressurization and friction, is

$$\frac{V_y^2}{2gH} = \frac{R^2}{1 - 2R^2}\left[1 - \frac{y}{H} - \left(1 - \frac{y}{H}\right)^{(1-R^2)/R^2}\right]. \quad (5)$$

For the ratio $R = 1/\sqrt{2}$, Eq. (5) becomes

$$\frac{V_y^2}{2gH} = \left(1 - \frac{y}{H}\right)\ln\left(\frac{1}{1 - (y/H)}\right). \quad (6)$$

As $1/\sqrt{2}$ is a very large area ratio, it is well represented in the early stage flow by free fall. An interesting feature of the flow, with no drain pipe length, is that the initial acceleration at which the level in the tank falls is always g regardless of the height of liquid in the tank or area of discharge.

The time required for the complete draining of the tank when $R = 1/\sqrt{2}$ is obtained bo integrating Eq. (6). The definite integral is

$$T = (H/\sqrt{2gH})\int_0^1 z^{-1/2}[-\ln z]^{1/2}\,dz = \left(\frac{H}{2g}\right)^{1/2}\frac{\Gamma(1/2)}{(1/2)^{1/2}} = \left(\frac{\pi H}{g}\right)^{1/2} \quad (7)$$

where $z = 1 - (y/H)$. It is found from the above equation that until about three-tenths of the tank has been drained the time-displacement relationship is practically that of free fall. The divergence becomes more significant as the draining of the tank nears completion. To complete the draining, 25% more time is required than the time it takes for a body to fall freely through this distance. Terminal flow in this case is not at all representative of the true state of affairs.

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DAVID BURGREEN
Nuclear Development Corporation of America
White Plains, New York
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Plastic Bending of Rods or Tubes with Radial Temperature Distributions

Horizontally loaded reactor fuel elements are subject to bending stresses which may limit the distance between fuel element supports. Since these fuel elements have radial temperature distributions, standard (*1*) methods of plastic analysis cannot be used to determine the deflections. This note contains the equations necessary to extend existing methods of plastic bending analysis to cases of rods or tubes with radial temperature distributions.

Assuming plane sections perpendicular to the neutral axis remain plane after bending, the equation relating the axial strain, ϵ, and the curvature, R (Fig. 1), is

$$\epsilon = \frac{z}{R} = \frac{r \sin\theta}{R}$$

where r is the distance from the center; z, the distance from the neutral axis; and θ, the angle between r and neutral axis.