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Letters to the Editors

Critical Equations for Finite Cylindrical Reactors

The construction of critical equations for reflected finite cylinder reactor cores presents an intrinsic difficulty. (Only thermal reactors with uniform transport properties throughout the core and all of an infinite reflector are considered in what follows. The core absorption cross section with the fuel absorption subtracted equals that of the reflector.) This arises from the necessity of satisfying the flux continuity boundary conditions at the two rims. The standard technique of matching interior (core) and exterior (reflector) eigenfunction expansions breaks down along these lines. Nuclear analysis of finite cylinder core, therefore, traditionally progresses through substitution of a so-called equivalent core having tractable geometry (sphere, slab, infinite cylinder). This process requires the prior assumption of an extrapolation distance.

Restatement of the problem in integral equation (and variational principle) form replaces the aforementioned difficulty with one of equal magnitude; to wit, the necessity for evaluating messy integrals over the finite cylinder. Use of the momentum space variety of variational principle recently introduced by Francis, et al. (*1*) is the key which permits evaluation of such integrals.

Consider the variational principle for the multiplication constant k expressed in momentum space:

$$[k] = \frac{\dfrac{1}{\sigma_r}\displaystyle\int |\Phi_t^-(\mathbf{P})|^2\, d\mathbf{P} - \dfrac{\sigma_t - \sigma_r}{\sigma_r}\displaystyle\int |\Phi_t^-(\mathbf{P})|^2 T(\mathbf{P})\, d\mathbf{P}}{\displaystyle\int |\Phi_t^-(\mathbf{P})|^2 \kappa(\mathbf{P})\, d\mathbf{P}} \tag{1}$$

$$\Phi_t^-(\mathbf{P}) = \int e^{-i\mathbf{P}\cdot\mathbf{r}} \psi_t(\mathbf{r})\, d\mathbf{r} \tag{2}$$

$$\sigma_r = \sigma_t/(1 - f). \tag{3}$$

Some slight rearrangement of Francis' result (*1*) is performed to obtain (1). The integrals in (1) extend over all of **P** space, and that in (2) only over the reactor core. The parameters σ_r and σ_t are macroscopic absorption cross sections for the core and reflector regions respectively while f is the thermal utilization for the core region. The transformed kernels $T(\mathbf{P})$ and $\kappa(\mathbf{P})$ are as defined in (*1*).

If the slowing-down kernel $\kappa(\mathbf{r})$ is assumed a convolution of Yukawa kernels, and the thermal diffusion kernel $T(\mathbf{r})$ is also assumed to have Yukawa form, then all the integrals in (1) involving kernels are linear combinations (*2*) of one integral form. Upon assuming a constant trial function

$$\psi_t(\mathbf{r}) = 1 \tag{4}$$

$$\Phi_t^-(\mathbf{P}) = (4\pi a/xy) \sin bx\, J_1(ay) \tag{5}$$

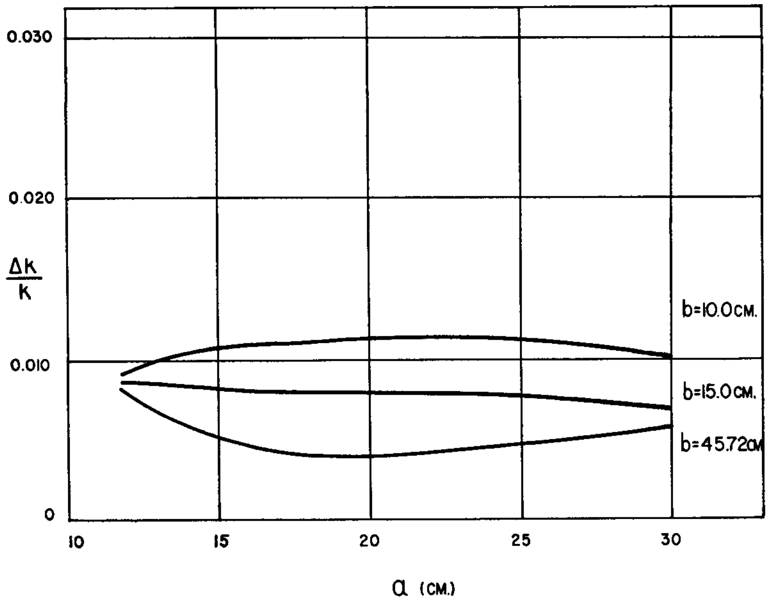

FIG. 1. $\Delta k/k$ *vs* a for various b; $\lambda = 6.0$ cm, $f = 0.90$.

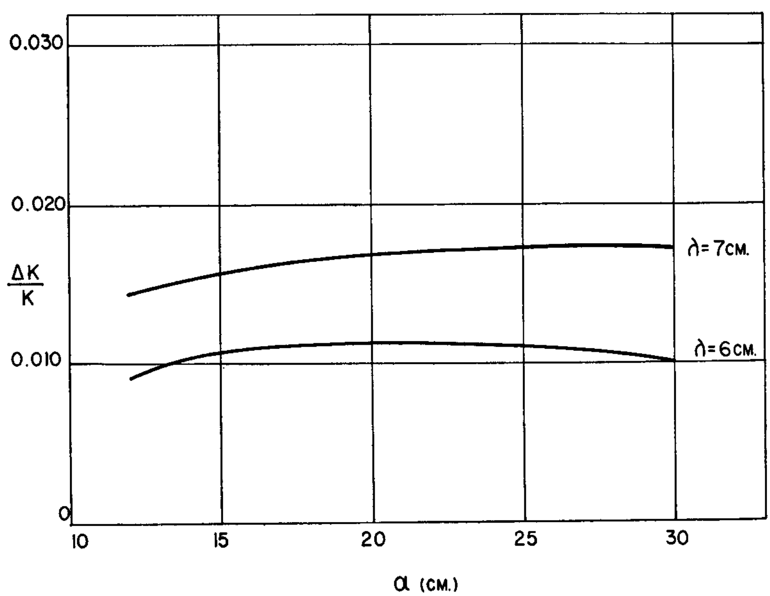

FIG. 2. $\Delta k/k$ *vs* a for various λ, $b = 10.0$ cm, $f = 0.90$.

this form becomes

$$I = \frac{8a^2b}{\sigma_t} \int_0^\infty dx \frac{\sin^2 x}{x^2} \int_0^\infty \frac{dy}{y} J_1^2(y) \frac{1}{1 + L^2[(y/a)^2 + (x/b)^2]} \cdot \tag{6}$$

The quantities a, b are core radius and half-height respectively while L is a group diffusion length. Similarly x, y are the axial and radial components of a cylindrical coordinate system in **P** space.

Equation (6) is approximately evaluated as

$$I = \frac{2\pi a^2 b}{\sigma_t} \left\{ 1 - \frac{L}{2b} + \frac{Le^{-2b/L}}{2b} - \frac{L}{a} + \frac{3L^3}{8a^3} + \frac{45L^5}{128a^5} - \frac{2L}{\pi a} K_1(2b/L) + \frac{2L^2}{\pi ab} \right.$$
$$\left. - \frac{L^2}{\pi ab} Ki_2(2b/L) - \frac{L^4}{\pi a^3 b} + O\left(\frac{L^4 e^{-2b/L}}{a^3 b}\right) + O\left(\frac{L^7}{a^7}\right) + \cdots \right\}, \tag{7}$$

In the above $J_n(z)$, $K_n(z)$ are the Bessel functons of standard definition and $Ki_n(z)$ is the transcendent discussed by Bickley (*3*). This result reduces to the previously derived forms (*2*) for an infinite cylinder ($b \to \infty$) or slab ($a \to \infty$) in the appropriate limit. The integration in (1) which does not involve a kernel is trivial.

The following situation is explored: A finite cylinder reactor core is light water moderated and reflected. Neutron slowing down is described by the three group synthetic kernel (*4*). The k required for criticality is calculated in the constant trial function approximation. An equivalent sphere radius R is next found by equating bucklings

$$\left(\frac{2.4048}{a + \lambda}\right)^2 + \left(\frac{\pi}{2(b + \lambda)}\right)^2 = \left(\frac{\pi}{R + \lambda}\right)^2 \tag{8}$$

The λ is an extrapolation distance. The k for this sphere is calculated (*2*), also in the constant trial function approximation, and compared with the finite cylinder k

$$\frac{\Delta k}{k} = \frac{k_{\text{cylinder}} - k_{\text{sphere}}}{k_{\text{cylinder}}} \cdot \tag{9}$$

In Fig. 1, (9) is plotted for a range of core sizes. In Fig. 2 the sensitivity of results to λ is examined. Note that the magnitude of $\Delta k/k$ appraises the validity of the sphericalization process.

An improved trial function would discriminate between that part of $\Delta k/k$ which is real and that part which reflects the inadequacy of (4). Inclusion of a parabolic term in the trial function, while increasing the tedious algebra many-fold, presents no major obstacle.

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