NUCLEAR SCIENCE AND ENGINEERING: **3**, 468–469 (1958)

LETTER TO THE EDITORS

An Iterative Method for Solving the P_l Equations in Slab Geometry

A great many reactor installations now make use of few-group diffusion codes, programmed for various machines. These codes, of course, are inadequate for classes of problems requiring higher approximations to the transport equation. It is possible, however, to do one-velocity P_3 or double-P_1 problems, in slab geometry, with the aid of a slightly modified diffusion code.

After some manipulation, the P_3 equations may be written in the form:

$$-\frac{d}{dx}D_1\left[\frac{d}{dx}\psi_1\right]+\Sigma_a\psi_1 = S + 2\Sigma_a\psi_2 \tag{1}$$

$$-\frac{d}{dx}\left[D_2\frac{d}{dx}\psi_2\right]+\left(\frac{5}{3}\Sigma_2+\frac{4}{3}\Sigma_a\right)\psi_2 = -\frac{2}{3}S+\frac{2}{3}\Sigma_a\psi_1 \tag{2}$$

where

$$\Sigma_l = \Sigma_a + \Sigma_s - \Sigma_{sl} \tag{3}$$

$$D_1 = \frac{1}{3\Sigma_1} \tag{4}$$

$$D_2 = \frac{3}{7\Sigma_3} \tag{5}$$

$$\psi_1 = F_0 + 2F_2 \tag{6}$$

and

$$\psi_2 = F_2\,. \tag{7}$$

Note that the source is assumed to be isotropic.

In the above equations, the Σ_{sl}'s are the usual Legendre polynomial coefficients of the differential cross section, while F_0 and F_2 are the zero-th and second Legendre polynomial coefficients of the directional flux.

Equations (1) and (2) may be interpreted as the two-group equations which determine the flux produced by a source in a multiplying medium. They may be solved by the standard iterative method which is normally used to compute reactivities. In brief, one gueses ψ_2 and solves Eq. (1) for ψ_1 . Equation (2) is then solved for a corrected ψ_2 , etc.

This iterative scheme will converge so long as the reactor or cell assembly described by Eqs. (1) and (2) as subcritical. It can be shown that the k_∞ for each material of the assembly is equal to

$$\left[\frac{1}{1+\frac{5}{4}\frac{\Sigma_2}{\Sigma_a}}\right] \leq \frac{4}{9}.$$

On physical grounds, it does not seem likely that a critical array can be constructed of such materials. Consequently the process should always converge and, in fact, one would expect convergence to be rapid.

The double $-P_1$ approximation (1) leads to equations formally identical with (1) and (2), after a redefinition of parameters. One finds again that

$$k_\infty = \left[\frac{1}{1+\frac{5}{4}\frac{\Sigma_2}{\Sigma_a}}\right]$$

with

$$\Sigma_2 = \frac{16}{15}\Sigma_T - \Sigma_{s2}.$$

Thus

$$k_\infty \leq \frac{3}{7}$$

with the equality sign holding if there is no scattering. Again one can make the same physical argument, which leads to the conclusion that the method should converge rapidly.

A variant of WANDA (2), (called SIMMPL) which solves the P_3 and double $-P_1$ equations has recently been debugged at Bettis. Problems are now being run on SIMMPL to assess the validity of these approximations.

Extension of this scheme to still higher P approximations leads to equations involving more groups. It has been proposed that these be solved by the Gauss-Seidel method. The convergence of the Gauss-Seidel method for P_5 and P_7 approximations is now under study. Preliminary results, obtained from a test code, indicate that the Gauss-Seidel method does converge.

Extension to other geometries is also being attempted.

REFERENCES

1. JOEL BENGSTON, ORNL 56-3-170 (1956).
2. ORVILLE J. MARLOWE *ET AL.*, WAPD-TM-28 (1956).

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Received February 10, 1958

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