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Letter to the Editor

Comment on the Similarity of the Neutron Fluxes in Simple Geometries

In treating the criticality problem for a homogeneous reactor, all standard textbooks on reactor physics arrive at a static one-group diffusion equation in the form

$$\nabla^2\phi(\boldsymbol{r}) + B^2\phi(\boldsymbol{r}) = 0 \tag{1}$$

or an equivalent of it. Introducing the concept of the extrapolation length, the boundary conditions to Eq. (1) are given as

$$\phi(\boldsymbol{r}_B) = 0 \ , \tag{2}$$

where $\boldsymbol{r}_B$ is the extrapolated boundary of the system. The treatment then usually continues to give solutions of Eq. (1), using Eq. (2), in an infinite slab, infinite cylinder, and sphere. These are given as

$$\phi(x) = A_1 \cos\left(\frac{\pi}{2a} x\right) ,$$

$$\phi(r) = A_2 J_0\left(\frac{2.405}{R} r\right) ,$$

and

$$\phi(r) = A_3 \frac{\sin\left(\frac{\pi}{R} r\right)}{r} , \tag{3}$$

with $x = \pm a$ and $r = R$ being the extrapolated boundaries of the slab, cylinder, and sphere, respectively. Normalization of the fluxes such that $\phi(0) = 1$ in Eq. (3) results in the solutions becoming rather similar qualitatively. This fact is usually illustrated by figures like Fig. 1; such figures can be found in a number of different textbooks on reactor physics.[1-11] Further, in this context, it is often stated that from the apparent similarity of the three curves, a cosine function can be used in all cases as a first approximation to the flux without incurring serious error.

A closer examination of the three curves reveals, however, that there is a significant difference between the slab and the other two cases, namely, that the solutions in the infinite cylinder and sphere exhibit an inflection point. A larger scale of the figure or any good plot of the functions $J_0(r)$ and $\sin(r)/r$, respectively, will readily show the inflection. This fact is usually not mentioned because the purpose is to demonstrate similarities and not differences. The only books that note some

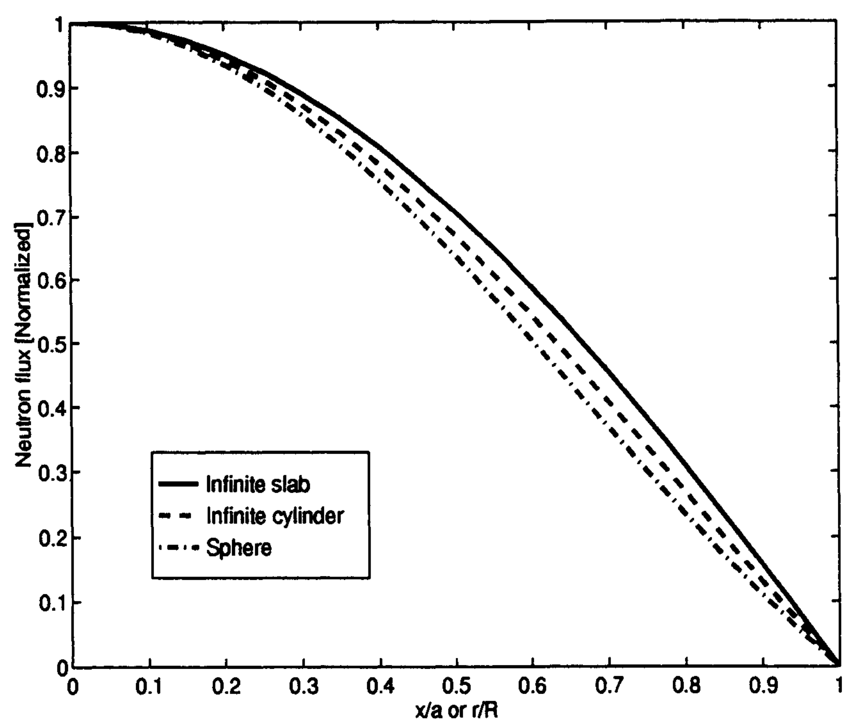

Fig. 1. The spatial variation of the neutron flux.

difference between the three solutions are those of Lamarsh[8] and Barjon.[10] They both note that the slopes of the fluxes, and thus the currents, are different at the boundary; the slope is the greatest for the slab and the smallest for the sphere. However, the presence of the inflection point constitutes a larger difference between the slab and the other two cases, which at a closer look should appear as somewhat unexpected. In addition, it may be embarrassing to the interested student who may discover the phenomenon but has no explanation for it. We shall therefore discuss it here.

The reason why we claim it should appear as unexpected is the following. In diffusion theory, the neutron current is given by Fick's law:

$$\boldsymbol{J}(\boldsymbol{r}) = -D\nabla\phi(\boldsymbol{r}) \ . \tag{4}$$

From Eq. (3) we see that the current in these simple geometries has only an x or an r component, respectively, so in the following, only those components will be considered. The inflection point in the space dependence of the flux in two and three dimensions means that the current is not monotonous. Thus, the current has a maximum at a point between the core center and the extrapolated boundary. This situation is illustrated in Fig. 2.

The intuitive expectation, of course, is that the current should increase monotonously from the center of the core to the boundary. This is because the current is given rise by the anisotropy of the flux. The anisotropy, on the other hand, increases monotonously from the center of the core [where the flux is completely isotropic, i.e., $\boldsymbol{J}(0) = 0$] toward the boundary, where it is maximum (no incoming neutrons at all). The current in the slab indeed exhibits this monotonous behavior in that it increases from zero at the center to a maximum at the boundary. At first sight, it would thus appear somewhat surprising that in two- and three-dimensional systems, this monotonous behavior is not found.

The explanation of the nonmonotonous behavior resides in the fact that what is loosely called the current in Eq. (4) is actually a density (current density). It is the net number of neutrons crossing a surface of unit area per unit time. For this reason, the International Organization for Standardization recommends the use of the denomination *particle flux density* for the flux and similarly the name *current density* should be used for the current.[12] Then, one observes that the total area of a surface (two planes, a cylinder, and a sphere around the origin, respectively), lying at a distance r from the core center and perpendicular to the current density, behaves proportionally to $r^{(N-1)}$ in an N-dimensional system. The current density can thus be written as the product of a true anisotropy factor $\boldsymbol{J}(\boldsymbol{r}) \cdot r^{(N-1)}$ and a geometry factor $1/r^{(N-1)}$. The anisotropy factor gives the total number of net neutrons crossing the surface. This is a monotonously increasing function of the distance from the origin (or proximity to the boundary) in all three dimensions, as shown in Fig. 3. The geometry factor, responsible for the dilution of the current density by the increase of the surface area, is a monotonically decreasing function of the distance from the origin in two and three dimensions. The resulting product of the two factors is nonmonotonous in two and three dimensions (Fig. 2).

The fact that the geometry factor leads to qualitatively different behavior in one dimension compared to higher dimensions is much better known from the solution of the diffusion equation in an infinite, homogeneous, nonmultiplying system with a unit plane (line, point) source. The solutions are

$$\phi(x) = \frac{L}{2D} e^{-|x|/L} \ ,$$

$$\phi(r) = \frac{1}{2\pi D} K_0\left(\frac{r}{L}\right) \ ,$$

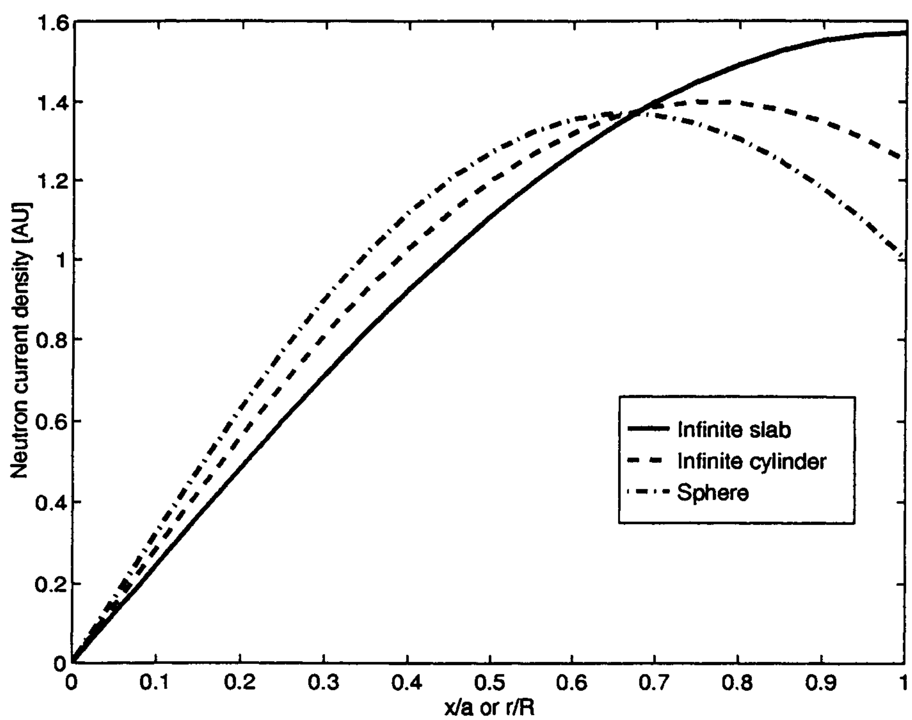

Fig. 2. The spatial variation of the neutron current density.

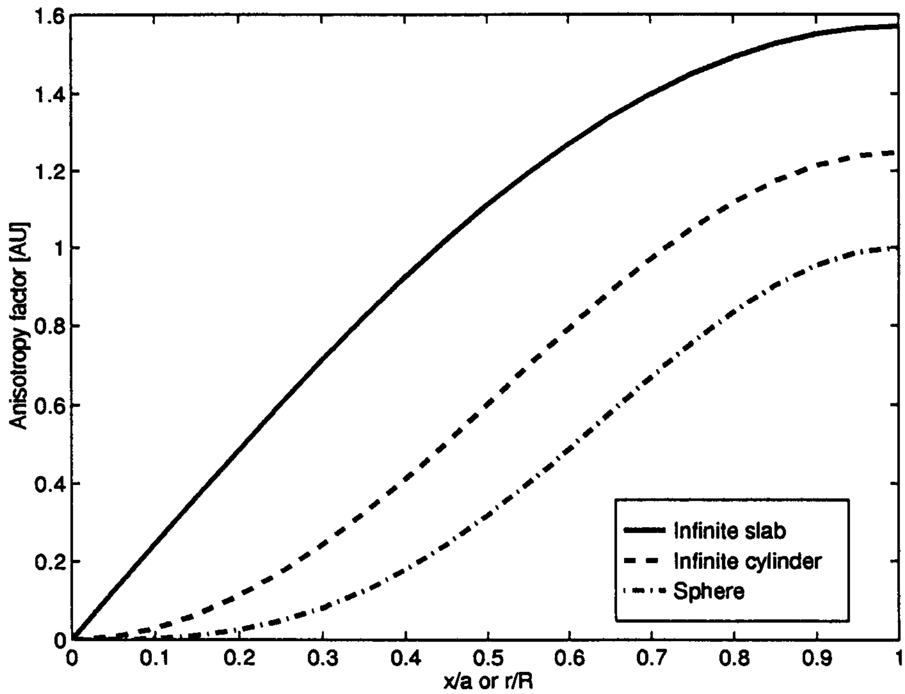

Fig. 3. The spatial variation of the anisotropy factor.

and

$$\phi(r) = \frac{1}{4\pi D} \frac{e^{-r/L}}{r} , \tag{5}$$

where L is the diffusion length and D is the diffusion coefficient. The solutions diverge at the origin in two and three dimensions, but not in one dimension. The reason is that shrinking a small volume in each dimension to the source, the total number of

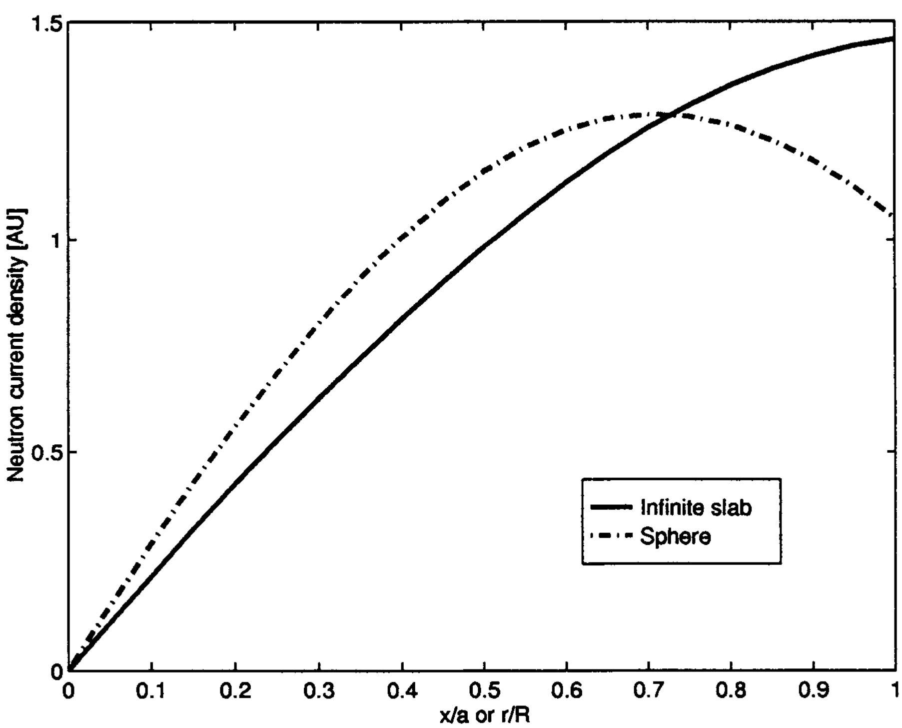

Fig. 4. The neutron current density as calculated from the transport equation for a slab and a sphere with 20 mean-free-path thickness and diameter, respectively.

particles crossing the surface must remain unity. In two and three dimensions, the surface area tends to zero; thus, the flux and current densities need to diverge. Again, the different dependence of the geometry factor on the distance from the origin leads to qualitatively different behavior in one and higher dimensions. The differences in this case are nevertheless quite large between the two classes (one and higher dimensions); hence, this fact is usually known. It was felt appropriate to point out a similar but much more subtle phenomenon in a source-free, critical, and finite system.

We note finally that the nonmonotonous behavior of the current density in higher dimensions is not a consequence of diffusion theory but exists also in transport theory. In Fig. 4, the dependence of the current density is shown for a slab and a sphere as calculated from an S_N solution of the transport equation with isotropic scattering.[13] The nonmonotonous behavior of the current density in the sphere is clearly seen.

Joakim Karlsson
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