Ref. 1 is concerned the following comments seem to be necessary.

On page 114 of Ref. 1 the statement stands: "One may also keep k_0 constant and allow P_{av} to vary according to Eq. (42). Since the magnitude and frequency of self-sustained oscillations are not subject to our control we must adopt this mode of operation in our stability analysis."

It should be reminded once more that there is no alternative, if the describing function technique is going to be used it is essential to assume that the initial power of the reactor is adjusted to a constant value. This corresponds to the assumption k_0 = constant, in Ref. 1. In this case P_{av} is a function of both magnitude and frequency of reactivity oscillations. It is, naturally, inherent in the statement self-oscillations (self-oscillations may be sustained or not) that the system can not have any varying input.

The analysis of the existence of a limit-cycle by the describing function technique is simply to check whether all the balance equations are satisfied for a certain magnitude and frequency of reactivity oscillations. Naturally, in the equations, all the functions must be known.

Equations (42) and (47) of Ref. 1 are respectively

$$k_0 + H(0)\,P_{av} = -2|x_1|^2\ Re\ Z_1$$

and

$$1 - P_{av}(\omega, |x_1|^2)\,H(i\omega)\,D_z(x_1, x_2, \omega) = 0 \quad .$$

There is also the relationship

$$P_0 = \frac{k_0}{|H(0)|}$$

which shows that, finally, the necessary assumption, P_0 = constant, has been made. The function P_{av}, evidently, affects the results derived from the balance equations as well as the function D_z. Any numeric result obtained or any statement made concerning the limit-cycle, in Ref. 1, is not acceptable because the function P_{av} is not known. If it is claimed that for a certain frequency and amplitude of reactivity oscillations there is a positive value of P_{av} which satisfies the equations, it has to be shown that the function P_{av} at oscillation frequency and amplitude assumes this value.

It seems also necessary to mention that the plot of the function

$$P_{av}(\omega, x_1^2)\,H(i\omega)\,D_z(x_1, x_2, \omega)$$

can not be called the Nyquist plot. Nyquist plot and Nyquist criterion concern only linear systems. While the Nyquist criterion relates the frequency domain behavior of a linear system to its time domain behavior, the describing function technique shows only the possibility of self-oscillations in the system. The results of such an analysis can not be used to derive conclusions concerning the general time-dependent behavior of a nonlinear system. Therefore, the statements: oscillation analysis by the describing function technique and the stability analysis by using the Nyquist criterion, can never be used interchangeably if the system is nonlinear.

The stability of the limit-cycle is a property mainly determined by the type of the describing function, not by the type of the equilibrium. If an unstable limit-cycle is perturbed inward or outward the trajectory of the perturbed motion moves away from the limit-cycle. A stable limit-cycle persists after any sufficiently small perturbation. But there is still a third category of the limit-cycle which is the semi-stable limit-cycle. Such a limit-cycle persists if perturbation is in one direction; for perturbations in the other direction the trajectory of the perturbed motion moves away from the limit-cycle. If equilibrium is stable and if the closest limit-cycle to the equilibrium is perturbed inward the trajectory of the perturbed motion reaches the origin, but this does not show that the limit-cycle is an unstable one. It can be a semi-stable limit-cycle. Besides the same system may have another stable limit-cycle outside the semi-stable one. A similar argument shows that the limit-cycle around an unstable equilibrium is not necessarily a stable one.

The dual-input describing function concept with respect to a nuclear reactor system has been discussed in Ref. 4. It is essential for the dual-input describing function to have two components. In the case of a low-power nuclear reactor, one component is defined to be used in the balance of the d.c. reactivity components, the other is defined to check the balance of the first-harmonic reactivity oscillations.

It has also to be noted that the purpose of defining a dual-input describing function for a low-power nuclear reactor has never been to increase the accuracy of the oscillation analysis. As stated in Ref. 4, the analysis is not possible by using only the first-harmonic describing function. To increase the accuracy of the analysis, balance of the second-harmonic reactivity oscillations may be taken into account, if a triple-input describing function of the low-power nuclear reactor is available. In this case the analysis becomes more complicated because of the increased number of balance equations. It is easy to realize that the technique loses all its practical significance when the number of components of the describing function is increased further.

Since the number of inputs of the describing function, regardless of how many, has to be finite, the nonlinear block must be followed by a low-pass filter for the analysis to be valid. Only in this case the error caused by the higher harmonics, the balance of which are not taken into consideration, can be negligible. Therefore, the idea in the statement by Akcasu et al.,[1] "We have presented a new concept in describing function analysis which no longer places the low pass filter restriction on the feedback in a reactor system," contradicts the well known describing function theory.

Sevim Tan

Nergis Sok. 15/17
Farabi, Cankaya
Ankara, Turkey

June 20, 1972

Reply to "Comments on The Concept of Multiple-Input Zero Power Describing Functions in Nuclear Reactors"

These remarks are directed to the preceding Letter by Tan.[1]

A. On the definition of describing function which is the real issue under discussion:

1. The question is whether we should define the describing function (a) with respect to the initial power P_0 (Tan's point of view) or (b) with respect to the average power, P_{av}, in the presence of periodic solution (the challenged definition). Denoting these two describing func-

[1]S. TAN, *Nucl. Sci. Eng.*, **49**, 405 (1972).

tions, respectively, by D_T and D_Z we pointed out in our paper (Ref. 2) that $D_T = (P_{av}/P_0)D_Z$, and that D_Z depends only on the amplitude and frequency of the sinusoidal input, whereas D_T, through the ratio of P_{av}/P_0, depends also on the initial phase of the input as well as on the manner in which the sinusoidal input is introduced, e.g., step, ramp, etc. Consequently, if we observe only P_{av} and the complex amplitude of the fundamental in the stationary power oscillations, we can uniquely measure D_Z, but D_T cannot be determined unless one specifies the initial conditions explicitly. In fact, the same stationary solution can be obtained in infinitely many ways starting with different P_0 each time and adjusting the initial conditions. For example, if Tan had used a cosine input in her *WKB* solution instead of a sine input, she would have obtained a different D_T. Conclusion: The dependence of D_T on the initial conditions makes it useless, if not meaningless, as an "intrinsic" reactor parameter. The experimental determination of D_Z by Wasserman (see Ref. 2) and the letter by Babala[3] are very pertinent to these discussions.

2. Performing an harmonic analysis in the presence of feedback, $H(i\omega)$, and ignoring the delayed neutrons one finds (see Ref. 2 and the references therein) that the limit cycles are determined by $1 - P_{eq}D_Z\ (\delta k, \omega)H(i\omega) = 0$ which contains the conventional describing function D_Z. This simple analysis proves definitely that it is D_Z rather than D_T that arises naturally in the nonlinear analysis of power oscillations. Although these arguments were presented in detail in Ref. 2, Tan does simply, and very conveniently, ignore them in her letter above.

3. In the presence of delayed neutrons, the average power $\overline{P}_{av}$ is different than the equilibrium level P_{eq} and, the characteristic equation is replaced by $1 - \overline{P}_{av}D_Z\ H = 0$. Somewhere in her letter, Tan suggests that we should "admit the error and call $\overline{P}_{av}D_Z$ the describing function..." If this were the only discrepancy, the issue would have been only a semantic one. But we still cannot interpret $\overline{P}_{av}D_Z$ as the zero-power describing function $D_\perp$, because although $P_0D_\perp = P_{av}D_Z$ the ratio (P_{av}/P_0) obtained as an initial value problem in the absence of feedback has nothing to do with $(\overline{P}_{av}/P_{eq})$. The former depends on initial conditions, whereas the latter depends only on the amplitude of the periodic oscillations. This difference seems to be consistently overlooked in Tan's work.

4. Concerning the function $P_{av}HD_Z$, Tan claims that it cannot be called a Nyquist plot. Any name is certainly acceptable. The real issue here is whether the plot of this function as a function of amplitude can give information about the stability of the limit cycle when they exist. We showed in an earlier paper[4] that the variation of the real part of this function with amplitude at $\omega = \omega_c$ and $k = k_c$ where the limit cycle occurs, gives information about the stability of the limit cycles. Therefore following the intersection of the plot with the real axis as a function of the amplitude one can investigate, at least on phenomenological grounds, the stability of the limit cycles. This approach was used in the above paper with application to various reactor types which displayed not only stable and unstable but also semi-stable limit cycles. It thus appears that the plot of $P_{av}HD_Z$ plays a similar role to that of the Nyquist plot in linear analysis. However, the theory needs a more careful study in this respect.

B. On the multiple-input zero-power describing function:

1. In our paper we have not considered the negative reactivity bias as an independent input because its value always depends on the other inputs to maintain periodic solution. Furthermore in the absence of delayed neutrons the reactivity bias is zero, and such an input is not needed. Our dual-input describing function would correspond to Tan's triple-input describing function.

2. Quoting only the first sentence in the summary of Ref. 2, Tan concludes that our idea of a describing function contradicts "the well-known describing function theory." Our idea is explained in the text of the paper as well as in the subsequent paragraph in the summary: It is known that the nonlinearity of the neutron kinetics generates harmonics as a response of a sinusoidal input in decreasing order in the reactivity amplitude δk. In fact the amplitude of the n'th harmonic is $A_n \sim P_0|Z_1 \ldots Z_n|(\delta k)^n$ where $Z_n = Z(i\omega n)$. Since $Z(i\omega)$ decreases as $(1/\omega)$ for large ω, the amplitudes of the higher harmonics decrease more rapidly than $(\delta k)^n$ as if a low-pass filter were introduced. If we desire a systematic perturbation analysis correct up to, say, the second order in δk, we keep track of the first and second harmonics in power as well as in reactivity without putting any restriction on the feedback transfer function. In this case, however, we use a dual-input describing function. Thus, the results can be made as accurate as needed by going to higher orders in the perturbation analysis in powers of δk without requiring an additional low-pass filter in the loop. Admittedly, such a scheme is impractical beyond $n = 2$, but we have carried the analysis in Ref. 2 for $n = 2$. Where is the contradiction?

In conclusion, Tan's letter does not contain new issues, beyond semantics, concerning the correct definition of describing function, to change our conclusions in Ref. 2, and to abandon a well accepted conventional definition of the describing function.

A. Z. Akcasu
M. Bost

The University of Michigan
Ann Arbor, Michigan 48108

July 5, 1972

[2] A. Z. AKCASU and C. M. BOST, Jr., *Nucl. Sci. Eng.*, **47**, 104 (1972).
[3] D. BABALA, *Nucl. Sci. Eng.*, **17**, 498 (1963).
[4] A. Z. AKCASU and L. M. SHOTKIN, *Nucl. Sci. Eng.*, **28**, 72 (1967).