Letters to the Editor

Comments on The Concept of Multiple-Input Zero Power Describing Functions in Nuclear Reactors

A paper recently published by Akcasu and Bost[1] claims to clarify the discrepancies between the different definitions of the describing function of a nuclear reactor. The paper, however, seems to confuse the issue rather than to clarify it. With reference to both analytical and experimental determination of a nuclear reactor describing function, it had to be stated clearly whether there were major errors, as indicated[2] in 1967, or not. It is evident that functions with major differences cannot all be called the describing functions of a nuclear reactor or of any other device. It has to be noted that the definition of the describing function can by no means contradict the linear theory. The following example is expected to clarify, if it is not yet clear, which one of the functions so far obtained is the "correct" describing function of a nuclear reactor.

A linear device with the transfer function

$$G(s) = \frac{1}{s(1+s)}$$

will be considered. Since the device is linear there will not be any difference between its frequency-response transfer function $G(j\omega)$ and its describing function $D(j\omega)$. Therefore

$$D(j\omega) = \frac{1}{j\omega(1+j\omega)} \quad .$$

The response of such a device to a sinusoidal input in the form $k \sin \omega t$, in the steady state, is

$$\frac{k}{\omega} + k\left|\frac{1}{j\omega(1+j\omega)}\right| \sin(\omega t + \phi) \quad ,$$

where

$$\omega = \text{Arg}\,\frac{1}{j\omega(1+j\omega)} \quad .$$

The gain of the describing function, as seen, can be obtained by dividing the amplitude of the sinusoidal component by k, which is the amplitude of the input. This is the "correct" gain. The amplitude of the output sinusoid divided by the d.c. component is

$$\omega\left|\frac{1}{j\omega(1+j\omega)}\right| \quad .$$

If this ratio is now divided by k the resulting function, which is

$$\frac{\omega}{k}\left|\frac{1}{j\omega(1+j\omega)}\right| \quad ,$$

has nothing to do with the gain. Therefore, if it is claimed that the function

$$\frac{\omega}{k} \times \frac{1}{j\omega(1+j\omega)}$$

is the describing function, the best counter comment would be to state that it is the "incorrect" describing function. It can neither be called the "convenient" describing function nor the "suitable" one. It is not even the "useful" describing function.

It seems also worthwhile to mention that in performing the sinusoidal test for a linear device, whose transfer function has a pole at zero, a d.c. bias may be used in the input in order to compensate the d.c. shift in the output. Naturally, if desired, the test can be performed at any constant d.c. level of the output. Because of the linear property of the device, the principle of superposition is valid. Therefore, the d.c. level of the output never affects the sinusoidal component.

The situation is, however, completely different in the case of a nonlinear device. If a purely sinusoidal input produces a d.c. shift in the output, as in the case of a low-power nuclear reactor, and if the d.c. level of the output is changed by using a d.c. bias, then the magnitude of output oscillations vary accordingly. The frequency data obtained this way will not all be on the same frequency characteristic. Therefore, as recommended,[2] first the power of the reactor must be carefully adjusted, then the sinusoidal reactivity input together with a d.c. bias, which is only sufficient to stabilize the output oscillations, is to be applied. The d.c. shift in the output must be recorded too, for each magnitude of the sinusoidal reactivity component, as a function of frequency, in order to obtain the dual-input describing function.[3] It is already shown that[4] without using the d.c. component, the oscillation analysis of a nuclear reactor system by using the describing function technique is not possible.

It is very interesting to note that although Ref. 1 has been written in a manner as if there was no error made in the past, every effort has been spent to correct it such that it would not affect the oscillation analysis. For example, Eq. (47) of Ref. 1 is

$$1 - P_{av}(\omega, |x_1|^2)\, H(i\omega)\, D_z(x_1, x_2, \omega) = 0 \quad . \qquad (1)$$

This equation will now be compared with

$$1 + D(j\omega)\, H(j\omega) = 0 \quad , \qquad (2)$$

which denotes a necessary condition for self-oscillations to start in a simple negative-feedback system. Naturally,

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[1] A. Z. AKCASU and C. M. BOST, Jr., *Nucl. Sci. Eng.*, **47**, 104 (1972).

[2] S. TAN, *Nucl. Sci. Eng.*, **30**, 436 (1967).

[3] J. C. HSU, A. U. MEYER, Modern Control Principles and Applications, pp. 244-249, McGraw-Hill Book Company Inc., New York, (1968).

[4] S. TAN, "The Dual-Input Describing Function in the Oscillation Analysis of a Nuclear Reactor System," IEEE *Trans. Nucl. Sci.*, NS-19, 2, 327 (1972).

in writing Eq. (2) it is assumed that the describing function technique is applicable. $D(j\omega)$ is the describing function of the nonlinear block; $H(j\omega)$ represents the frequency-response transfer function of the feedback block which is assumed to be linear. As seen, if D_z is claimed to be the describing function the major difference between Eq. (1) and Eq. (2) is that Eq. (1) consists of a third function which is $P_{av}(\omega,|x_1|^2)$. The existence of such a function in the balance equation is not consistent with the describing function technique. Therefore, it would be more reasonable to admit the error and call $P_{av}D_z$ the describing function instead of trying to alter the theory to cope with an incorrect function.

It is also to be noted that in the low frequency region the argument of the function D_z is completely different from the argument of the transfer function (Fig. 3, Ref. 1). The describing function obtained by the *WKB*-method,[2] however, reveals that there is not an appreciable difference between the argument of the describing function and the argument of the transfer function. For these inconsistent results, the procedure used in the determination of D_z has to be blamed. The results in this procedure have been obtained by assuming a form for the amplitude-series in the complex Fourier expansion of power. The validity of such an assumption, however, is not known. Without having a convergence problem, the amplitudes in the series expansion of the logarithm of flux have already been obtained by the *WKB*-method (Eq. 16, Ref. 2). It is seen that, even with very small reactivity magnitudes if the frequency is also very small, in the logarithm of flux, large perturbations occur. In such a case it is hard to expect a correct result by determining only one or two terms of a series whose convergence is not known. Therefore, before calculating the magnitude and phase of D_z as a function of frequency, for δk = 0.5 dollar, it was necessary for Akcasu et al.[1] to prove the convergence of the series. Under the conditions it seems that it was a waste of time to try to investigate the effects of the second and higher harmonics of reactivity upon the fundamental of the power oscillations, by using the same approach. Besides, this approach does not yield the function $P_{av}(\omega,|x_1|^2)$. Therefore, neither the over-all describing function of the feedback system shown in Fig. 1, Ref. 1, can be determined nor the oscillation analysis of the same system can be performed. The function $P_{av}D_z$ which identifies the nonlinear block must be completely determined initially.

At this stage it seems worthwhile to mention that the feedback system which consists of two noninteracting blocks, one being the low-power nuclear reactor, the other being an external feedback block can not be called the power reactor. This fact will be explained as follows. Assume a power reactor is represented by the following equations:

$$\frac{dn}{dt} = \frac{\rho - \beta}{l^*} n + \lambda C \quad , \tag{3}$$

$$\frac{dC}{dt} = \frac{\beta}{l^*} n - \lambda C \quad , \tag{4}$$

$$\rho = \rho_{ex} - \rho_f \quad , \tag{5}$$

$$\rho_f = K(n - n_0) \quad . \tag{6}$$

Here, K is a positive constant. ρ_{ex} is the external component of reactivity and n_0 represents the equilibrium power or the equilibrium neutron density. From these four equations the neutron density n may be obtained as a function of ρ_{ex}. Assume this relationship to be

$$n = f_1(\rho_{ex}) \quad . \tag{7}$$

Now, if Eqs. (5) and (6) are ignored, from Eqs. (3) and (4) the relationship

$$n = f_2(\rho) \tag{8}$$

will be obtained. This, evidently, describes the dynamic behavior of a low-power nuclear reactor as a function of ρ which is the external reactivity. Equations (3) and (4) represent a linear system. Because of this reason the describing function of a low-power nuclear reactor has been determined first.[2] It would, naturally, be very convenient if it were possible to obtain the describing function of a power reactor, using the describing function of a low-power reactor, the equation $\rho = \rho_{ex} - \rho_f$ and the linear relationship between ρ_f and n. It must be observed, however, that, if ρ is substituted by $\rho_{ex} - \rho_f$ in Eq. (8), which describes the dynamic behavior of a low-power nuclear reactor, the equation

$$n = f_2(\rho_{ex} - \rho_f)$$

is obtained. This is, evidently, not equivalent to

$$n = f_1(\rho_{ex})$$

which is the equation of the power reactor, except for very specific cases.

The substitution of ρ by $\rho_{ex} - \rho_f$ corresponds to the assumption that the power reactor can be represented by the block diagram shown in Fig. 1. Then such an assumption is not valid.

If the feedback system shown in Fig. 1 is in a control circuit, as Fig. 2 indicates, without using its over-all describing function the oscillation analysis can be made, because, in this case, instead of the two linear blocks, one equivalent feedback block can be defined. That is, everything would be relatively simple if a low-power nuclear reactor with external feedback were representing a power reactor.

As far as the search of limit-cycle as explained in

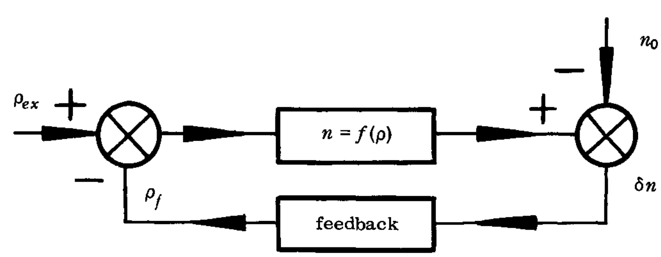

Fig. 1. A low-power nuclear reactor with external feedback.

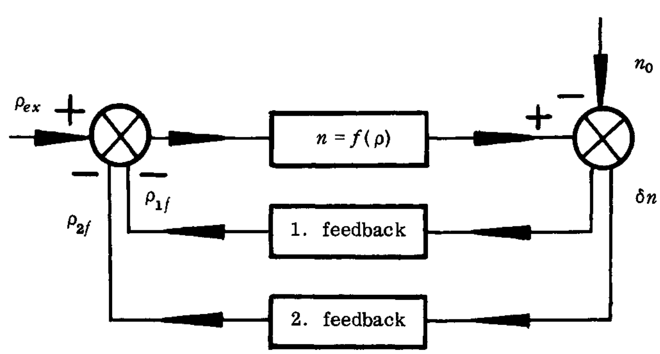

Fig. 2. A low-power nuclear reactor with external feedback in a different control circuit.

Ref. 1 is concerned the following comments seem to be necessary.

On page 114 of Ref. 1 the statement stands: "One may also keep k_0 constant and allow P_{av} to vary according to Eq. (42). Since the magnitude and frequency of self-sustained oscillations are not subject to our control we must adopt this mode of operation in our stability analysis."

It should be reminded once more that there is no alternative, if the describing function technique is going to be used it is essential to assume that the initial power of the reactor is adjusted to a constant value. This corresponds to the assumption k_0 = constant, in Ref. 1. In this case P_{av} is a function of both magnitude and frequency of reactivity oscillations. It is, naturally, inherent in the statement self-oscillations (self-oscillations may be sustained or not) that the system can not have any varying input.

The analysis of the existence of a limit-cycle by the describing function technique is simply to check whether all the balance equations are satisfied for a certain magnitude and frequency of reactivity oscillations. Naturally, in the equations, all the functions must be known.

Equations (42) and (47) of Ref. 1 are respectively

$$k_0 + H(0)\,P_{av} = -2|x_1|^2\; Re\; Z_1$$

and

$$1 - P_{av}(\omega, |x_1|^2)\,H(i\omega)\,D_z(x_1, x_2, \omega) = 0 \quad .$$

There is also the relationship

$$P_0 = \frac{k_0}{|H(0)|}$$

which shows that, finally, the necessary assumption, P_0 = constant, has been made. The function P_{av}, evidently, affects the results derived from the balance equations as well as the function D_z. Any numeric result obtained or any statement made concerning the limit-cycle, in Ref. 1, is not acceptable because the function P_{av} is not known. If it is claimed that for a certain frequency and amplitude of reactivity oscillations there is a positive value of P_{av} which satisfies the equations, it has to be shown that the function P_{av} at oscillation frequency and amplitude assumes this value.

It seems also necessary to mention that the plot of the function

$$P_{av}(\omega, x_1^2)\,H(i\omega)\,D_z(x_1, x_2, \omega)$$

can not be called the Nyquist plot. Nyquist plot and Nyquist criterion concern only linear systems. While the Nyquist criterion relates the frequency domain behavior of a linear system to its time domain behavior, the describing function technique shows only the possibility of self-oscillations in the system. The results of such an analysis can not be used to derive conclusions concerning the general time-dependent behavior of a nonlinear system. Therefore, the statements: oscillation analysis by the describing function technique and the stability analysis by using the Nyquist criterion, can never be used interchangeably if the system is nonlinear.

The stability of the limit-cycle is a property mainly determined by the type of the describing function, not by the type of the equilibrium. If an unstable limit-cycle is perturbed inward or outward the trajectory of the perturbed motion moves away from the limit-cycle. A stable limit-cycle persists after any sufficiently small perturbation. But there is still a third category of the limit-cycle which is the semi-stable limit-cycle. Such a limit-cycle persists if perturbation is in one direction; for perturbations in the other direction the trajectory of the perturbed motion moves away from the limit-cycle. If equilibrium is stable and if the closest limit-cycle to the equilibrium is perturbed inward the trajectory of the perturbed motion reaches the origin, but this does not show that the limit-cycle is an unstable one. It can be a semi-stable limit-cycle. Besides the same system may have another stable limit-cycle outside the semi-stable one. A similar argument shows that the limit-cycle around an unstable equilibrium is not necessarily a stable one.

The dual-input describing function concept with respect to a nuclear reactor system has been discussed in Ref. 4. It is essential for the dual-input describing function to have two components. In the case of a low-power nuclear reactor, one component is defined to be used in the balance of the d.c. reactivity components, the other is defined to check the balance of the first-harmonic reactivity oscillations.

It has also to be noted that the purpose of defining a dual-input describing function for a low-power nuclear reactor has never been to increase the accuracy of the oscillation analysis. As stated in Ref. 4, the analysis is not possible by using only the first-harmonic describing function. To increase the accuracy of the analysis, balance of the second-harmonic reactivity oscillations may be taken into account, if a triple-input describing function of the low-power nuclear reactor is available. In this case the analysis becomes more complicated because of the increased number of balance equations. It is easy to realize that the technique loses all its practical significance when the number of components of the describing function is increased further.

Since the number of inputs of the describing function, regardless of how many, has to be finite, the nonlinear block must be followed by a low-pass filter for the analysis to be valid. Only in this case the error caused by the higher harmonics, the balance of which are not taken into consideration, can be negligible. Therefore, the idea in the statement by Akcasu et al.,[1] "We have presented a new concept in describing function analysis which no longer places the low pass filter restriction on the feedback in a reactor system," contradicts the well known describing function theory.

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June 20, 1972

Reply to "Comments on The Concept of Multiple-Input Zero Power Describing Functions in Nuclear Reactors"

These remarks are directed to the preceding Letter by Tan.[1]

A. On the definition of describing function which is the real issue under discussion:

1. The question is whether we should define the describing function (a) with respect to the initial power P_0 (Tan's point of view) or (b) with respect to the average power, P_{av}, in the presence of periodic solution (the challenged definition). Denoting these two describing func-

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[1]S. TAN, *Nucl. Sci. Eng.*, **49**, 405 (1972).