Letters to the Editor

Comments on the Effect of Transverse Dimensions on the Diffusion Length in Crystalline Moderators

Ahmed, Kothari, and Kumar[1] have recently criticized the author's work[2-4] on the behavior of the neutron diffusion length in systems with finite transverse dimensions. In particular, the work in Ref. 4 is said to depend on the assumption of space-energy separability. We deny this allegation and refer the reader to Eq. (5) of Ref. 4; viz:

$$\Psi_\nu(x, E) = \tfrac{1}{2} \int_{-a}^{a} dx' \, K_\nu(|x - x'|; E) \int_0^\infty dE' \, \Sigma(E' \to E) \times \Psi_\nu(x', E') \quad , \tag{1}$$

where the finite medium diffusion length ν^{-1} is defined by

$$\phi(E, x, z) \sim e^{-\nu z} \, \Psi_\nu(x, E) \quad , \tag{2}$$

K_ν is a kernel defined in Ref. 4 and the situation refers to the decay along the z axis in a slab of width '$2a$'.

The equation for $\Psi_\nu(x, E)$ is exact and its solution does not involve space-energy separability but rather the determination of the so-called K integrals and subsequent use of a variational method to determine the appropriate form of the transverse buckling. Indeed, the solution in the transverse direction is shown to take the following form:

$$\Psi_\nu(x, E) = C_0 \, M(E) \, g_0(E) \cos B_x x - M(E) \, h_0(x, E) \quad , \tag{3}$$

where B_x^2 is the transverse buckling and $M(E)go(E)$ is the spectrum characterizing the asymptotic part of the solution. As stated in Ref. 4, this spectrum is independent of the transverse dimensions and is identical with the infinite medium solution. Contrary to the statement by Ahmed, Kothari, and Kumar, it was *not* "found that the energy dependence of the total flux is independent of the transverse dimensions"; the statement was made that the asymptotic part of the total flux is unaffected by transverse leakage. Quite clearly, the transient term $M(E) \, h_0(x, E)$ will cause the total flux to depend on transverse dimensions.

The theory expounded in Ref. 4 refers only to the case when a unique exponential decay exists; that is,

$$\nu^2 = \kappa^2 + B_x^2 \quad , \tag{4}$$

where B_x^2 is the transverse buckling and $1/\kappa$ is the infinite medium diffusion length. By definition, $1/\kappa$ is independent of transverse dimensions and no contradiction exists in Ref. 4. Ahmed, Kothari, and Kumar are therefore incorrect when they state that "the experimental results of DeJuren and Swanson are not explained." Indeed they are explained for the case when the exponential decay exists. When it does not exist, the theory in Ref. 4 does not claim to explain the results. What it does do, however, is to give an accurate criterion for the critical transverse dimensions at which exponential decay ceases. In view of the inevitable uncertainty involved in experimental work of determining precisely when exponential decay does cease, the criterion is in excellent agreement with the work of DeJuren and Swanson.

As for the case of nonexponential decay, we have (Ref. 3) discussed this problem at some length and have explained by the use of a simple scattering model how the asymptotic flux disappears and the angle-energy spectrum becomes a nonseparable function of space, energy, and angle [see Eq. (30) of Ref. 3]. Some further comments on the behavior of the flux when no exponential decay exists are given in Refs. 5 and 6; moreover, the limitations of the asymptotic approximation method are also fully discussed there.

One puzzling aspect of the paper by Ahmed, Kothari, and Kumar is in their interpretation of the infinite medium diffusion length $1/\kappa$. They remark that, for transverse dimensions greater than 80×80 cm^2 in graphite, a unique diffusion length K_1 appears to exist; this is close to the author's own estimate of 96×96 cm^2.[4] However, they then state that $1/\kappa$ varies with size in the range 200×200 cm^2 to 70×70 cm^2. Whilst a variation from 80×80 cm^2 downward can be understood as a pseudo-asymptotic effect, it is difficult to comprehend the infinite medium diffusion length itself varying with size; surely this is by definition independent of size. The explanation of this anomalous situation lies in the adoption of an energy dependent buckling which prevents the ansatz

$$\Psi(\boldsymbol{\nu}, E) = \cos\{B_x(E)x\} \cos\{B_y(E)y\} \, \phi(E) \, e^{-Kz} \tag{5}$$

being a true solution of the diffusion equation. This form of solution can, in fact, be regarded only as a useful representation of the flux and can never correspond to the true solution which we know should be space-energy separable deep inside the medium (when K is unique). Yet another way of interpreting Eq. (5) is to write it in the slab case; i.e.,

$$\Psi(x, E) = \cos\{B_x(E)x\} \, \phi(E) \, e^{-Kz} \tag{6}$$

and compare it with the correct form of the solution given by Eqs. (2) and (3). We see then that the energy dependent buckling constitutes an attempt to introduce a correction factor into the asymptotic solution to account for the

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[1] FEROZ AHMED, L. S. KOTHARI, and ASHOK KUMAR, *Nucl. Sci. Eng.*, **46**, 203 (1971).
[2] M. M. R. WILLIAMS, *Nukleonik*, **9**, 305 (1967).
[3] M. M. R. WILLIAMS, "Existence of a Diffusion Length in a Finite Prism of a Pure Moderator," *Proc. Symp. Neutron Thermalization and Reactor Spectra*, Vol. 1, p. 27, International Atomic Energy Agency (1967).
[4] M. M. R. WILLIAMS, *Nukleonik*, **11**, 219 (1968).
[5] M. M. R. WILLIAMS, *J. Math. Phys.*, **9**, 1873 (1968).
[6] M. M. R. WILLIAMS, *J. Math. Phys.*, **9**, 1885 (1968).

neglected transient term. This criticism can be applied to the pulsed neutron problem which has also been treated by the energy dependent buckling concept.[7]

To give due credit to the idea of an energy dependent buckling, it must be admitted that it does constitute a practical, if unsophisticated, method of solving problems in finite geometry and generally yields results in reasonable accord with experiment. It is questionable, however, whether it provides a fundamental understanding of the basic physical processes involved and, moreover, coupled with diffusion theory it is unlikely to be of value in the region below the Bragg cutoff where the mean-free-path is of the same order as, or greater than, a characteristic transverse dimension. For example, the mean-free-path in graphite at the Bragg cutoff energy is about 20 cm.

Finally, it is the author's opinion that the only satisfactory method of dealing with problems of this type is through the transport equation analyzed in terms of its natural eigenfunctions. In this way, fundamental parameters such as κ and B are uniquely defined and can be obtained directly from experiment. Any other procedure, however effective it may be in reproducing the experimental results, can only be viewed as an artifice.

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[7]F. AHMED and A. K. GHATAK, *Nucl. Sci. Eng.*, **33**, 106 (1968).

Reply to "Comments on the Effect of Transverse Dimensions on the Diffusion Length in Crystalline Moderators"

In the preceding Letter Williams[1] has commented upon our recent paper, "Effect of Transverse Dimensions on the Diffusion Length of Neutrons in Crystalline Moderator Assemblies".[2] He criticizes our interpretation of his work and also our method of solving the problem. We would like to comment on these two aspects separately:

1. While studying diffusion of neutrons in an assembly with transverse buckling ($B_\perp^2$) less than the critical buckling (B_c^2), one is normally interested in the decay of the fundamental (or asymptotic) mode. According to Williams this asymptotic flux is space-energy separable. For example, a little beyond Eq. (5) of his letter he says ". . . the true solution which we know should be space-energy separable deep inside the medium (when K is unique)." (We do not agree with this and will comment on it a little later). Since we were *not* interested in transients, when we talked of neutron flux it was in relation to the fundamental model (or pseudo-asymptotic mode when $B_\perp^2$ was greater than B_c^2) and as such we did not misquote Williams.

It is true that in his paper, Williams[3] starts with a very general form for the solution of the Boltzmann equation, yet his conclusions and final results have been deduced for the asymptotic part of the flux, which, according to him, is space-energy separable. In his variational approach he also uses a trial function which is space-energy separable.

Williams certainly gives "an accurate" criterion for the critical transverse dimensions at which exponential decay ceases. However, when we stated that he does not explain the DeJuren and Swanson results,[4] we were talking about the variation of K with buckling. We still maintain that he does not explain this explicitly.

2. We agree that the ansatz [Eq. (5) of Williams' Letter] that we have used is not an *exact* solution of the Boltzmann equation and we have said so in our paper. There, we have argued at some length that for small assemblies energy-dependent boundary conditions would be physically more appropriate. Once this is granted, our ansatz is a very good lowest order solution. We have shown that the terms we neglect are a few orders of magnitude smaller than those that we retain. (Along the axis of the assembly our solution is exact.) One frequently uses a similar approach in many branches of physics, quantum mechanics being one of them.

Our values of κ^2 (ν^2 in the notation of Williams) are buckling dependent and, in general, we cannot obtain infinite medium diffusion from this by subtracting some suitable buckling. What we have shown is that in the limit of $B_\perp^2 = 0$, our definition of κ^2 reduces to the standard definition. As such Williams remarks in the paragraph just above Eq. (5) are therefore inapplicable.

We are aware of the fact that the mean-free-path of cold neutrons is very large compared to that of neutrons in the thermal energy range. As mentioned in our paper, comparative studies of the solutions of the general transport equation, and of this equation under diffusion approximation, have been made by many workers (more recently by Nishina[5]). They generally find that diffusion theory gives results that are in close agreement with those obtained by transport theory, even in regions of parameters where diffusion theory would normally be expected to fail. Hence our use of the diffusion approximation is not unjustified.

We do not understand why Williams thinks that "*the* satisfactory method of dealing with *problems of this type* is through the transport equation analyzed in terms of its natural eigenfunctions." The problem that we pose—that of solving the Boltzmann equation in the diffusion approximation with energy dependent boundary conditions—is a clearly defined problem and, we feel, a physically more realistic one. Ours is the first attempt to solve it and we hope better solutions will soon be forthcoming.

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[1]M. M. R. WILLIAMS, *Nucl. Sci. Eng.*, **47**, 498 (1972).

[2]FEROZ AHMED, L. S. KOTHARI, and ASHOK KUMAR, *Nucl. Sci. Eng.*, **46**, 203 (1971).

[3]M. M. R. WILLIAMS, *Nukleonik*, **11**, 219 (1968).

[4]J. A. DeJUREN and V. A. SWANSON, *J. Nucl. Energy*, **20**, 905 (1966).

[5]K. NISHINA, "Energy Dependent Diffusion Theory Treatment of Neutron Wave Propagation in a Finite Medium," PhD Thesis, The University of Michigan (1969).