Book Review

Advanced Mathematical Methods for Scientists and Engineers. By Carl M. Bender and Steven A. Orszag. McGraw-Hill Book Company (1978). 593 pp. $29.95.

Advanced mathematics books seem to fall into two main categories. The first, epitomized by the splendid four-volume series *Methods of Modern Mathematical Physics* by Mike Reed and Barry Simon, is devoted to a rigorous development of the principles underlying mathematical physics. This means, primarily, functional analysis, which is to say the theory of operators in topological spaces (mostly Banach spaces).

The second category is devoted to a description of problem-solving methods, with somewhat less attention to rigorous development. This type of book is well exemplified by the *magnum opus* of Morse and Feshbach, *Methods of Theoretical Physics.*

In my school days, scientists and engineers were rarely if ever exposed to the functional analytic approach, and so we learned all of our mathematics in a problem-solving context. Now times have changed somewhat, and many scientists and engineers are learning functional analytic techniques, if not from Reed and Simon, from other excellent texts, for example, Kreyszig's more elementary treatment *Introductory Functional Analysis with Applications* or perhaps the more abstract *Functional Analysis* by Bachman and Narici. (The reader will pardon me for naming my personal favorites. There are many other excellent treatises available—Riesz-Nagy, Akhiezer-Glazman, Taylor, Edwards, etc.)

Applied scientists who do study functional analysis ought to supplement their knowledge with at least a brief study of problem-solving methods. For this purpose, the text of Morse and Feshbach is no longer appropriate for a number of reasons. First, it is too long and detailed to serve a supplementary role. Second, it is not written for students already skilled in abstract functional analytic techniques, who might be somewhat dismayed by its lack of rigor and bored by its classical approach. Finally, it is some 30 years old, and so does not treat a number of topics of current burning interest.

The text of Bender and Orszag under review seems to me ideal to fill the need described above. The presentation is clear, the examples well chosen, the treatment and topics are modern, and the analysis sufficiently rigorous not to offend the purist, but also sufficiently down-to-earth not to frighten anyone.

There are 11 chapters in the book. Topics include the basic theory of linear and nonlinear differential and difference equations; methods of approximate solutions to the same; asymptotic expansions of integrals and sums, with particular application to special functions, and a nice treatment of saddle point methods; perturbation theory, with special attention to the method of matched asymptotic expansions; summation of series, with an analysis of the important topic of Padé approximates; boundary layer theory, the WKB method and multiple-scale analysis (which includes the WKB and boundary layer approximations as special cases, and is especially important in problems of nonlinear stability).

Unlike the Morse and Feshbach text, that of Bender and Orszag requires a previously acquired grounding in complex analysis (as, indeed, do the Reed-Simon volumes). An advanced undergraduate course at the level of Marsden's *Basic Complex Analysis* should suffice nicely.

Every chapter is introduced by an appropriate quotation from Arthur Conan Doyle. For example, Chap. 5, "Approximate Solutions of Difference Equations," quotes Dr. Watson (from *A Study in Scarlet*) as saying "What ineffable twaddle! I never read such rubbish in my life." (The relevance of this particular quotation is a wee bit obscure.)

Although my earlier remarks have stressed the use of the Bender-Orszag text as a supplement to a Reed-Simon type approach, those whose knowledge and use of mathematics are strictly at the applied level can also use this text with profit and enjoyment. I heartily recommend this textbook to all scientists and engineers involved in any sort of analytical work. As a graduate text, I feel strongly that it should follow a course in functional analysis, but not because functional analysis is a prerequisite to understanding the material, only because the understanding will be motivated at a deeper level. I am sure many, many graduate students in science and engineering will profitably use this book as their one and only venture into higher mathematics.

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About the Reviewer: Paul Zweifel returns to these columns with another careful review reflecting his extended experience as an academic, presently being the University Distinguished Professor of Physics and Nuclear Engineering and director of the Laboratory for Transport Theory and Mathematical Physics at the Virginia Polytechnic Institute and State University. Dr. Zweifel's career has been broadened by association with industrial and government-operated laboratories. His own studies began at Carnegie Tech and Duke.