Letters to the Editor

Comments on the Bayesian Method for Estimating Reactor Core Melt Frequency

In a recent paper, Apostolakis and Mosleh[1] present a Bayesian analysis for the estimation of reactor core melt frequency. Martz[2] has already pointed out how Apostolakis and Mosleh inexplicably reversed the role of data and prior opinion. An unfortunate error in Martz's Letter[2] (since corrected[3]) obscured examination of their numerical results. However, because of the significance of this application, it is important to scrutinize the basis of their analysis and the meaning of the results.

The problem considered is the estimation of the nuclear power plant core melt rate, λ. The posterior probability distribution arrived at by the authors, their "bottom line," was quite similar to the results of Ref. 4. Thus, Rasmussen, who directed the Reactor Safety Study (RSS), was able to go before Congress and cite as support for the results of the RSS, the Bayesian analysis by Apostolakis and Mosleh, which "account(ed) for the Browns Ferry fire, the WASH-1400 results, the opinions of several groups including the Union of Concerned Scientists and the Environmental Protection Agency," as well as "310 years of (U.S.) reactor experience with no core melts."[5] Let us examine this accounting.

The analysis by Apostolakis and Mosleh has two components:

1. a "likelihood function" for λ based on WASH-1400 results and criticism of those results
2. a prior distribution for λ based on 310 reactor-yr marred only by Browns Ferry.

These are multiplied to yield a posterior distribution. In deriving a likelihood function, it is the authors' assessment that the WASH-1400 modal estimate of λ, namely $\lambda^* = 1.5 \times 10^{-5}$, is most likely an underestimate of λ by a factor of 10. Thus, their likelihood function, which is a gamma function, has a mode at $\lambda_m = 1.5 \times 10^{-4}$. Also, 95% of their likelihood function falls below $\lambda_{95} = 7.1 \times 10^{-4}$ (this spread is arrived at from a consideration of critiques of Ref. 4).

It is helpful to express the parameters of the likelihood function in terms of pseudo-data. In this case, the authors' likelihood function corresponds to pseudo-data of one core melt in 6667 reactor-yr. That is, it is the authors' assessment after considering Ref. 4 and its critics that the likelihood function for λ is the same as the Poisson likelihood function based on one core melt in 6667 reactor-yr. (Though this seems an optimistic rendering, we'll let it stand as the authors' personal assessment, but not as an "objective" accommodation of the critics' opinions.)

In deriving a prior distribution, the authors seek to make use of the available data. After some involved reasoning, they decide to count the Browns Ferry fire as 0.03 core melts. Given the pseudo-data of 1/6667, it is clear that adding in the additional "data" of 0.03 core melts in 310 reactor-yr, by any method, should have a very small effect. For example, by conventional methods, one obtains a joint likelihood function that is maximized at $\lambda = 1.03/6977 = 1.48 \times 10^{-4}$, which is negligibly different from the initial 1.5×10^{-4}. However, Apostolakis and Mosleh,[1] after modifying their likelihood function by their prior distribution, obtained a modal value of $\lambda_m = 1.8 \times 10^{-5}$ and an upper 95% posterior probability bound of 5×10^{-4}, both of which are quite close to WASH-1400 results. That is, the authors' finding is that the meager 0.03/310 data nearly offset the fairly strong assessment that WASH-1400 was most likely off by a factor of 10. Based on the available information, such a conclusion doesn't seem justified. The question is how did the authors obtain their results?

The answer lies in their choice of prior distribution. This distribution was to represent the information embodied in 310 reactor-yr of operation marred only by the Browns Ferry fire. The authors arrived at a gamma prior distribution with parameters $\alpha = 0.12$ and $\beta = 120$. In terms of pseudo-data this prior amounts to -0.88(!) core melts in 120 reactor-yr, a result that is difficult to interpret. In deriving their prior distribution, the authors wanted a distribution with a mean of $\langle\lambda\rangle = 0.03/310 = 10^{-4}$ and a 99th percentile of 1.5×10^{-2} (which happens to be the upper 99% statistical confidence limit on λ based on zero core melts in 310 yr) and the one they arrived at has a mean of 10^{-3} and a 99th percentile of 1.5×10^{-2}. As a result, they thought that they had made a conservative choice for a prior. Unfortunately, such was not the case.

When the two sets of pseudo-data are pooled, the result is a function (which one can think of as a joint likelihood function or a posterior density), which is maximized at $\lambda_m = [1 + (-0.88)]/(6667 + 120) = 1.8 \times 10^{-5}$, the authors' result. The single core melt of the likelihood function has been nearly wiped out by the negative 0.88 core melts of the prior distribution.

Bayesian methods have considerable appeal as a method of merging data, opinion, and other information. However, without due care, these methods can result in a misrepresentation of available information, as appears to be the case under discussion.

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[1] G. APOSTOLAKIS and A. MOSLEH, *Nucl. Sci. Eng.*, **70**, 135 (1979).

[2] H. F. MARTZ, Jr., *Nucl. Sci. Eng.*, **72**, 368 (1979).

[3] H. F. MARTZ, Jr., *Nucl. Sci. Eng.*, **74**, 158 (1980).

[4] "Reactor Safety Study," WASH-1400, U.S. Nuclear Regulatory Commission (1975).

[5] Statement of Norman C. Rasmussen before the subcommittee on Energy and the Environment, House Committee on Interior and Insular Affairs (Feb. 26, 1979).