Void Streaming in S_N Calculations

Corrections for the end effects of neutron streaming through void regions generally are expressed[1-3] in terms of critical bucklings or effective migration areas and diffusion coefficients. These are then applied when solving the diffusion equation in the plane normal to the direction of streaming (e.g. in the horizontal midplane of a reactor with vertical void channels). While this is convenient in diffusion theory, it is not appropriate in transport theory since D, the diffusion coefficient, does not appear explicitly. Conversely, one should utilize the angular variable in the transport solution and express the directional streaming losses explicitly.

Correction for these transverse streaming losses in one- or two-dimensional transport solutions is simple when the S_N method is used[4].

Let the region dV (Fig. 1) be a differential volume element of the void region within the three-space X-Y-Z. We wish to solve the two-dimensional transport equation in the plane X-Y or the one-dimensional transport equation in the direction $\hat{x}$ (plane geometry) or $\hat{r}$ (cylinder geometry). In each situation, particle loss by streaming occurs in the direction $\hat{z}$.

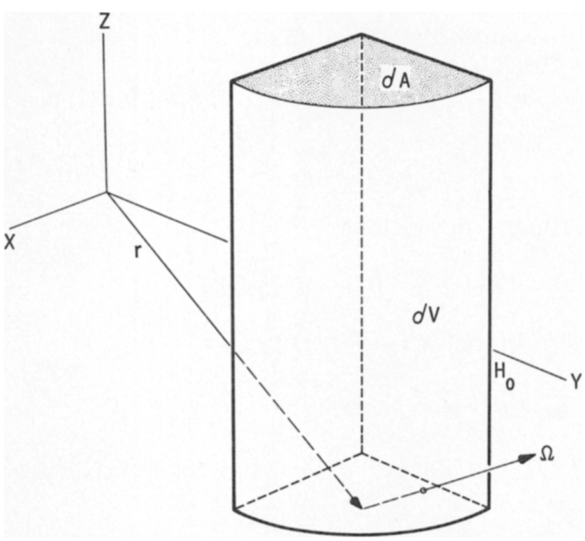

Fig. 1. Geometry of neutron streaming from void region.

Let H_0 be the half-height or extent of the void region above the symmetry plane X-Y. The term N is the number of particles flowing in the direction Ω at position r, for each unit time, and for each unit area normal to the direction Ω.

In the X-Y geometry instance, the leakage rate in direction Ω from the volume dV is $\hat{z}\cdot\Omega NdA$ but the leakage rate for each *unit* volume is $\hat{z}\cdot\Omega NdA/dV$ or $\frac{\hat{z}\cdot\Omega}{H_0} N$.

Thus the transport equation becomes

$$\Omega\cdot\nabla N + \left(\Sigma + \frac{\hat{z}\cdot\Omega}{H_0}\right)\bar{N} - \bar{S} = -\frac{\partial N}{\partial t}\,. \tag{1}$$

In the void region the collision cross section Σ and the source term S are zero. Outside the void region the term $\frac{\hat{z}\cdot\Omega}{H_0}$ is omitted. Equation (1) is obtained similarly in the one-dimensional plane or cylinder geometry case.

Including the void correction $\frac{\hat{z}\cdot\Omega}{H_0}$ in the solution of Eq. (1) is simple when using the S_N method since the factor $\hat{z}\cdot\Omega$ already appears explicitly or is readily available.

In the 2DXY-plane[5] and DSN-cylinder[6] solution it is

$$\hat{z}\cdot\Omega = \bar{\mu}_m\;, \tag{2}$$

where $\bar{\mu}_m$ is the linearly averaged value of $\hat{z}\cdot\Omega$ in the m-th polar angle segment[4]. In the DSN plane-geometry solution it is

$$\hat{z}\cdot\Omega = \sqrt{1-\bar{\mu}_m^{\,2}}\;, \tag{3}$$

where $\bar{\mu}_m$ is the linearly averaged value of $\Omega\cdot\hat{x}$ in the m-th angular segment[4]. The newer S_N codes[7], *DTK*, *DDK*, and *DTF*, can be modified similarly to implement the correction.

The void correction term in Eq. (1) is an approximation, appropriate only for general (two-dimensional) and degenerate (one-dimensional) solutions in the plane X-Y. In a three-dimensional representation the leakage probability is a function of the Z coordinate also; a particle traveling in some direction $\Omega \neq \hat{Z}$ obviously has a higher leakage probability near the ends ($Z = \pm H_0$) of the void region than at the center ($Z = 0$): it has

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[5]J. BENGSTON *et al.*, "2DXY, Two-Dimensional Cartesian Coordinate S_N Transport Calculation," AGN-TM-392, Aerojet-General Nucleonics (1961).

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[7]W. J. WORLTON, personal communication (August 1963).

a smaller chance of crossing the void gap and entering the adjacent non-void regions.

Therefore, one should undertake some numerical checks of the method described here before applying it to specific problems. I do not have an opportunity to do this, at present, and have submitted this letter trusting someone else will.

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Comments on "Generalizations of Fick's Law"*

In this letter we present several comments on a letter by Kostin[1]. For brevity, we shall refer to "Eq. (n) of Ref. 1" as "(K-n)."

(I) The "generalized Fick's law" (K-11) is alleged to have the simplicity of Fick's law. We believe that this is not so, for the following reasons: 1) The quantity which appears under the derivative sign is $\overline{v^2}(z)N(z)$ and not $N(z)$. 2) The $\overline{k}_{tr}(z)$, which plays the role of a reciprocal diffusion coefficient, is not an intensive property of the medium[2,3]. Even in a homogeneous medium, $\overline{k}_{tr}(z)$ may vary with z. 3) the $\overline{k}_{tr}(z)$ depends explicitly on the neutron distribution $n(z,v,\underset{\sim}{\omega})$. In the time-dependent form (K-7), the $\overline{k}_{tr}(z,t)$ depends explicitly on $n(z,v,\underset{\sim}{\omega},t)$. It follows that $\overline{k}_{tr}(z,t)$ may be a function of time in a system with fixed composition and fixed boundaries.

(II) The definition (K-5) of $\alpha(z,v')$ is, with trivial changes in notation,

$$\int \Sigma_s(z,v'\rightarrow v, \underset{\sim}{\omega}'\cdot\underset{\sim}{\omega}) v dv d\omega = v'\alpha(z,v')\Sigma_s(z,v').$$

This definition does not lead to (K-7) or (K-11); however a corrected definition

$$\int \Sigma_s(z,v'\rightarrow v,\underset{\sim}{\omega}'\cdot\underset{\sim}{\omega}) v\mu dv d\omega = v'\mu'\alpha(z,v')\Sigma_s(z,v') \quad (1)$$

does lead to (K-7) and (K-11). (In an appendix at the close of this letter, we answer a question regarding the definition of α in Eq. (1).) To verify this statement, apply the operator $\int \mu v dv d\omega$ to (K-4) to obtain

$$\frac{\partial J(z,t)}{\partial t} = -\frac{\partial}{\partial z}\int \mu^2 v^2 n(z,v,\underset{\sim}{\omega},t) dv d\omega - \int \mu v^2\,\Sigma(z,v)\,n(z,v,\underset{\sim}{\omega},t) dv d\omega + \int \mu v v'\,\Sigma_s(z,v'\rightarrow v,\underset{\sim}{\omega}'\cdot\underset{\sim}{\omega}) \times n(z,v',\underset{\sim}{\omega}',t)\,dv'd\omega' dv d\omega\ . \quad (2)$$

By use of the definition of $\alpha(z,v')$ from Eq. (1) we may write the final term as

$$\int \mu'(v')^2\,\alpha(z,v')\,\Sigma_s(z,v')\,n(z,v',\underset{\sim}{\omega}',t) dv'd\omega'$$

$$= \int \mu v^2\,\alpha(z,v)\,\Sigma_s(z,v) n(z,v,\underset{\sim}{\omega},t) dv d\omega$$

and obtain (K-7) and (K-11) unaltered. The definition (K-5) does not allow this transformation of the final term of Eq. (2).

(III) If the goal is to remove the one-velocity restriction from (K-1), then we feel that a better way is to apply the operator $\int \mu d\omega$ rather than $\int v\mu dv d\omega$ to (K-4). Furthermore, we feel that it is preferable to include on the right-hand side of (K-4) a source term $S(z,v,\underset{\sim}{\omega},t)$. This source may be caused by fissions, and we assume it to be isotropic in the sense that $\int \mu S d\omega \equiv 0$. In the time-independent case, the immediate result of applying $\int \mu d\omega$ to this modified (K-4) is

$$0 = -v\frac{\partial}{\partial z}\int \mu^2 n(z,v,\underset{\sim}{\omega}) d\omega - v\Sigma(z,v)\int \mu n(z,v,\underset{\sim}{\omega}) d\omega + \int \mu v' n(z,v',\underset{\sim}{\omega}')\Sigma_s(z,v'\rightarrow v,\underset{\sim}{\omega}'\cdot\underset{\sim}{\omega}) dv'd\omega' d\omega.$$

With the definitions

$$J(z,v) = v\int \mu n(z,v,\underset{\sim}{\omega}) d\omega\ ,$$

$$N(z,v) = \int n(z,v,\underset{\sim}{\omega}) d\omega\ ,$$

$$\Sigma_s(z,v'\hookrightarrow v) = \int \Sigma_s(z,v'\rightarrow v,\underset{\sim}{\omega}'\cdot\underset{\sim}{\omega}) d\omega\ ,$$

$$\int \mu\Sigma_s(z,v'\rightarrow v,\underset{\sim}{\omega}'\cdot\underset{\sim}{\omega}) d\omega = \mu'\overline{\mu_0}(z,v'\rightarrow v)\Sigma_s(z,v'\rightarrow v), \quad (3)$$

and

$$\overline{\mu^2}(z,v) = \int \mu^2 n(z,v,\underset{\sim}{\omega}) d\omega / N(z,v)\ ,$$

this result becomes

$$0 = -v\frac{\partial}{\partial z}[\overline{\mu^2}(z,v)N(z,v)] - \Sigma(z,v)J(z,v) + \int J(z,v')\overline{\mu_0}(z,v'\rightarrow v)\,\Sigma_s(z,v'\rightarrow v) dv'\ . \quad (4)$$

(In an appendix at the close of this letter, we give concise answers to certain questions regarding the definition of $\overline{\mu_0}$ in Eq. (3).)

*Work performed under the auspices of the USAEC.

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