TABLE I

Transport Averages

Parameter	Carbon	Water
$\bar{\lambda}_t$	0.9995 λ_0	1.074 λ_0
$\frac{\overline{\lambda_t v}}{\bar{v}}$	0.9917 λ_0	1.218 λ_0
$(\lambda)_{\bar{v}}$	0.9975 λ_0	1.093 λ_0

The second average, $\frac{\overline{\lambda_t v}}{\bar{v}}$, is also the flux-weighted average of λ_t. In the case of carbon the averages agree within one percent.

the material and leakage (not the geometric) bucklings, $B_e^2 = -\frac{1}{L^2} + \frac{2\pi^2}{a^2}$; however, this expression has not been derived rigorously.

Probably the best approach would be to correct all the diffusion lengths directly to the room-temperature spectrum. The diffusion equation, $D_0\nabla^2 n - n v\Sigma_a = 0$ can also be corrected for spectral changes and becomes

$$(D_0 - CB_e^2)\nabla^2 n - n v\Sigma_a = 0 ,$$

$$\nabla^2 n - \frac{n}{L^2} = 0 ,$$

$$L^2 = \frac{D_0 - CB_e^2}{v\Sigma_a} = \frac{D_0\left(1 - \frac{CB_e^2}{D_0}\right)}{v\Sigma_a} = L_0^2\left(1 - \frac{CB_e^2}{D_0}\right)$$

where L_0 is the diffusion length for a Maxwellian spectrum at room temperature. For infinite geometry, where it is agreed $B_e^2 = -\frac{1}{L^2}$, the usual relationship

$$L^2 = L_0^2 + \frac{C}{D_0} = \frac{D_0}{v\Sigma_a}\left(1 + \frac{v\Sigma_a C}{D_0^2}\right)$$

can be obtained.

In addition, the extrapolation length measurements exhibited a reasonably constant behavior as a function of copper poison and are inconsistent with a 5.4% decrease in λ_t because of diffusion heating.

At Hanford, diffusion length measurements have been made in a heated graphite stack coupled to a thermal column[3]. Since

$$L^2 = \frac{D_0}{v\Sigma_a} = \frac{\overline{\lambda_t v}}{3\bar{v}\Sigma_a} ,$$

L^2 should vary as $\bar{v}$ or $T^{\frac{1}{2}}$ if λ_t and $\overline{v\Sigma_a}$ are constant. The best fit to the measurements was $L^2 \sim T^{0.48 \pm 0.01}$ or $\lambda_t \sim T^{-0.02 \pm 0.01}$. In the poison experiments, if λ_t is assumed constant, $\bar{v} = \bar{v}_0\left(1 - \frac{C}{D_0}B_e^2\right)$ where $\bar{v}$ is the actual mean velocity of a stack and $\bar{v}_0$ is the Maxwellian averaged velocity at room temperature. In the most heavily poisoned case the mean velocity would be about 0.3% greater than the room temperature average, or the effective temperature 0.6% greater. (For an infinite geometry the effective temperature is 1.0% greater than ambient.) This will cause a negligible change in λ_t. It should be pointed out that the poison measurements in carbon permit determination of λ_t by two equations: (a) $\Delta X = 0.7104\ \lambda_t$ where ΔX is the extrapolation distance and (b) $D_0 = \frac{\overline{\lambda_t v}}{3}$ where $\frac{1}{D_0}$ is the slope of the $\frac{1}{L^2}$ versus Σ_a poison curve. The extrapolation value of 2.74 ± 0.03 cm is in good agreement with the value derived from the copper-poison D_0. The pulsed D_0 values, however, are consistently low.

For carbon, using $\lambda_t = \lambda_0\left(\frac{T}{T_0}\right)^{-0.02}$, averages involving λ_t are given over a Maxwellian spectrum,

$$n(T)dT = \frac{2\pi}{(\pi T_0)^{\frac{3}{2}}}\, e^{-\frac{T}{T_0}}\, T^{\frac{1}{2}}\, dT . \qquad (3)$$

For comparison, averages are given for water at a density of 1.0 g/cm^3 where $L^2 \sim T^{0.87}$ or $\lambda_t = \lambda_0(T/T_0)^{0.37}$, is assumed[4] and λ_0 is the transport mean free path at 0.025 eV. The transport mean free path at the mean velocity, $(\lambda)_{\bar{v}}$ is also included.

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[3]R. C. LLOYD, E. D. CLAYTON and C. R. RICHEY, *Nucl. Sci. Eng.* 4, 5, 690-697 (1958).

[4]M. REIER and J. A. DeJUREN, *Reactor Sci. Tech.* (J.N.E. Parts A/B) 14, 18-24 (1961).

Reply to Remarks by J. A. DeJuren on "A Note on the Measurement of the Transport Mean Free Path of Thermal Neutrons in Graphite by a Poison Method"

There are at least five methods by which the transport mean free path of thermal neutrons, λ_t,

in graphite has been measured or inferred. It was my intent in a letter to the editor[1] to demonstrate that the pulsed-neutron method and the poison method yield consistent values, as they should, since the (λ, B_g^2) space in the pulsed-neutron experiment can be considered as an extension of the $(\Sigma_a, 1/L^2)$ space in the poison experiment. This agreement is to be expected because both methods rely primarily upon diffusion theory, with similar assumptions, inferences and restrictions applying to both cases. I assumed λ_t to be the Maxwellian average of the transport mean free path in the infinite medium, while the CB^4 term corrects for deviations from Maxwellian spectrum in the finite medium.

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[1]G. A. PRICE, "A Note on the Measurement of the Transport Mean Free Path of Thermal Neutrons in Graphite by a Poison Method," *Nucl. Sci. Eng.*, 18, pp. 400-413, (1964).

A Monte Carlo Technique for Selecting Neutron Scattering Angles from Anisotropic Distributions*

The angular distributions of elastically scattered neutrons become anisotropic in the center-of-mass system at neutron energies above about 100 keV, the degree of anisotropy increasing with neutron energy. For many problems in neutron transport at such energies, a satisfactory solution is obtained by taking account of the anisotropy is only an approximate fashion. Diffusion theory, for example, accounts approximately for anisotropy by the use of the transport cross section, $\Sigma - \bar{\mu}\Sigma_s$, where Σ is the total cross section, Σ_s is the elastic scattering cross section, and $\bar{\mu}$ is the average cosine of the laboratory scattering angle. For some problems, however, a more accurate treatment of anisotropic scattering is required. Calculations of neutron distributions deep within thick shields or, at the other extreme, in very small critical systems are examples of such problems.

When the Monte Carlo method is used to solve neutron-transport problems, angular distributions can be included to as high a degree of accuracy as desired; however, in general, the higher the degree of accuracy demanded the more costly the solution, since the selection of a scattering angle becomes an elaborate procedure and requires a large amount of computer time. A selection method is presented here which gives the same accuracy as that obtained by a straightforward selection from a Legendre expansion but requires considerably less computer time.

Express the distribution function, $F(\mu)$, of the cosine of the scattering angle, μ, as the Legendre series

$$F(\mu) = \sum_{\ell=0}^{n} \frac{2\ell+1}{2} f_\ell P_\ell(\mu) + \sum_{\ell=n+1}^{\infty} \frac{2\ell+1}{2} f_\ell P_\ell(\mu), \tag{1}$$

where

$$f_\ell = \int_{-1}^{1} F(\mu) P_\ell(\mu)\, d\mu, \tag{2}$$

but only the first sum is known, while the sum of the remainder of the terms from $n+1$ to infinity is unknown. Consider a second distribution function $G(\mu)$ given by

$$G(\mu) = \sum_{k=0}^{n} \phi_k \delta(\mu - \theta_k) \quad . \tag{3}$$

Expanding the δ-function gives

$$G(\mu) = \sum_{k=0}^{n} \phi_k \left\{ \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_\ell(\theta_k) P_\ell(\mu) \right\}$$

$$= \sum_{\ell=0}^{n} \frac{2\ell+1}{2} \left\{ \sum_{k=0}^{n} \phi_k P_\ell(\theta_k) \right\} P_\ell(\mu) + \tag{4}$$

$$+ \sum_{\ell=n+1}^{\infty} \frac{2\ell+1}{2} \left\{ \sum_{k=0}^{n} \phi_k P_\ell(\theta_k) \right\} P_\ell(\mu) \, .$$

Now if one sets

$$f_\ell = \sum_{k=0}^{n} \phi_k P_\ell(\theta_k) \, , \tag{5}$$

Eq. 4 and Eq. 1 are identical; hence they are good to the same order of approximation when both are truncated at n.

To find the ϕ_k, multiply Eq. 5 by $\frac{2\ell+1}{2} P_\ell(\theta_j)$ and sum over ℓ from 0 to n

$$\sum_{k=0}^{n} \phi_k \sum_{\ell=0}^{n} \frac{2\ell+1}{2} P_\ell(\theta_k) P_\ell(\theta_j) = \sum_{\ell=0}^{n} \frac{2\ell+1}{2} f_\ell P_\ell(\theta_j). \tag{6}$$

As shown in Appendix A,

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