Moderator Scattering and Flux Depression Near Resonances*

The first-order solution for the flux near an intermediate-width resonance has been obtained by Goldstein and Cohen[1] in terms of a width parameter λ, and a second-order flux obtained by iteration. It has been pointed out by Goldstein[2] that this iterate and all higher iterates done in the same manner will fail to show the correct reduction in flux on the low-energy side of the resonance. Only use of the Placzek solution as a Green's Function will produce this reduction in a rigorous way. While this is correct in the strictest sense, it is possible to obtain such a reduction in an *ad hoc* but reasonable manner by treatment of scattering by the moderator in the same approximation as heavy-element scattering.

The general second-order iterate is:

$$\psi_\lambda^{(2)} = \frac{1}{s+\sigma}\left(s\,K_m\,\psi_\lambda^{(1)} + K\sigma_s\,\psi_\lambda^{(1)}\right) , \qquad (1)$$

where s is the moderator cross section for each absorber atom; K_m is the slowing-down integral operator for the moderator; and K is the corresponding operator for the absorber. The notation generally follows that of Refs. 1 and 2. By rearranging $\psi_\lambda^{(1)}$

$$\psi_\lambda^{(1)} = \frac{1+X^2}{\beta_\lambda^2 + X^2} = 1 - \frac{1+\lambda\dfrac{\Gamma_n}{\Gamma_\gamma}}{s+\lambda\sigma_p}\,\sigma_a\,\psi_\lambda^{(1)}$$

and inserting into Eq. (1), it becomes

$$\psi_\lambda^{(2)} = \frac{1}{s+\sigma}\left[s+\sigma_p - \frac{1+\lambda\dfrac{\Gamma_n}{\Gamma_\gamma}}{s+\lambda\sigma_p} \times \right.$$

$$\left. \times\left(s\,K_m\,\sigma_a\,\psi_\lambda^{(1)} + \sigma_p\,K\sigma_a\,\psi_\lambda^{(1)}\right) + K\sigma_{sr}\,\psi_\lambda^{(1)}\right] . \qquad (2)$$

The expression

$$s\,K_m\,\sigma_a\,\psi_\lambda^{(1)} + \sigma_p\,K\sigma_a\,\psi_\lambda^{(1)}$$

becomes, when the integrals are performed,

$$e^{u-u_r}\,I_\lambda^{(1)}\left[\frac{1}{1-\alpha_m}\left(\tan^{-1}\frac{X+\delta_m}{\beta_\lambda} - \tan^{-1}\frac{X}{\beta_\lambda}\right) + \right.$$

$$\left. + \left(\frac{1}{1-\alpha}\right)\left(\tan^{-1}\frac{X+\delta}{\beta_\lambda} - \tan^{-1}\frac{X}{\beta_\lambda}\right)\right] .$$

This has the form of the first-collision term in a Placzek solution and repeated applications of the scattering operator would be expected to lead to the asymptotic solution. If this term is replaced, *ad hoc*, by the asymptotic one, the flux becomes

$$\psi_\lambda^{(2)} = \frac{1}{s+\sigma}\left[s+\sigma_p - \frac{I_\lambda}{\xi}\left(\frac{1}{2} - \frac{1}{\pi}\tan^{-1}\frac{X}{\beta_\lambda}\right) - \right.$$

$$- \left(\frac{1+\lambda\dfrac{\Gamma_n}{\Gamma_\gamma}}{s+\lambda\sigma_p} - 1\right)\left(sK_n\sigma_a\,\psi_\lambda^{(1)} + \sigma_p\,K\sigma_a\psi_\lambda^{(1)}\right) +$$

$$\left. + K\sigma_{sr}\,\psi_\lambda^{(1)}\right] . \qquad (2a)$$

In the limit of $X \to -\infty$ this goes over into

$$\lim_{X\to -\infty}\ \psi_\lambda^{(2)} = 1 - \frac{I_\lambda}{\xi(s+\sigma_p)} = p_\lambda^{(1)} ,$$

the desired result.

For most situations, the error in the resonance integral caused by neglection of the effect of the flux depression on moderator scattering is small, being on the order of $\frac{1}{2}(1-p)$.

It should also be mentioned that there is a certain problem of logical consistency involved in the iteration operation, since the expression adopted for the first-order flux implies that the cross section for slowing down by the absorber is $\lambda\,\sigma_s$ rather than σ_s .

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[1] R. GOLDSTEIN and E. R. COHEN, "Theory of Resonance Absorption of Neutrons," *Nucl. Sci. Eng.*, **13**, 132-140 (1962).

[2] R. GOLDSTEIN, "Spectral Distribution of Neutrons in the Vicinity of a Resonance," *Trans. Am. Nucl. Soc.*, **7**, 1, p. 28 (1964), and *Nucl. Sci. Eng.* **19**, 359-362 (1964).