First-Collision Probability of an Infinite Rectangular Cylinder

The first-collision probability for a medium of arbitrary convex shape containing neutrons born isotropically is given[1] by

$$P_c = 1 - (4\pi V)^{-1}\int dS \int (1-e^{-R_s})\,\underset{\sim}{\Omega}\cdot \underset{\sim}{n_i}\, d\Omega\,, \quad (1)$$

where the notation is that of Ref. 1 except that all distances are in units of mean free paths. For any point g on the surface of the infinite cylinder, the intersection of a plane constructed through $\underset{\sim}{\Omega}$ and $\underset{\sim}{n_i}$ with the cylinder is a rectangle, as shown in Fig. 1. The distance fg is denoted by A, B is the distance ef, and ϕ is the angular coordinate in the surface containing point g. In the plane efg, $\underset{\sim}{\Omega}\cdot\underset{\sim}{n_i} = \mu$ and integration over μ gives

$$P_c = 1-(4\pi V)^{-1}\int dS \int \{1/2 - E_3[A] - E_3[B] + $$
$$+ E_3[(A^2+B^2)^{\frac{1}{2}}]\}\, d\phi\,, \quad (2)$$

where $E_n(x)$ are the tabulated exponential functions[1]

$$E_n(x) = \int_0^1 y^{n-2}\, e^{-x/y}\, dy. \quad (3)$$

Simplifying Eq. (2), the first-collision probability for an infinite cylinder of rectangular cross section with side lengths a_1 and a_2 is

$$P_c = -1 + M_{1,2} + M_{2,1} + Q_c(a_1) + Q_c(a_2)\,. \quad (4)$$

In Eq. (4), $Q_c(a_i)$ is the first-collision probability for a slab[1] of thickness a_i and $M_{i,j}$ is defined as

$$M_{i,j} = (\tfrac{1}{2}\pi a_1 a_2)^{-1}\int_0^{a_i} dx \int_0^{\frac{1}{2}\pi} \{E_3[x/\cos\phi] -$$
$$- E_3[\sqrt{(x/\cos\phi)^2 + a_j^{\,2}}]\}\, d\phi\,,\ i,j = 1,2\,. \quad (5)$$

Numerical evaluation of Eq. (4) was performed for $a_1 = a_2 = a$ and results are shown in Fig. 2. The differences in first-collision probabilities between the square cylinder and various approximations[2] are defined as

$$D_1 = R_c\,(2a\pi^{-\frac{1}{2}}) - P_c(a)$$
$$D_2 = P_c(a) - R_c(a)$$
$$D_3 = P_c(a) - [Q_c(a)]^2\,, \quad (6)$$

where $P_c(a)$ is the first-collision probability for the square cylinder and $R_c(a)$ is the probability for a circular cylinder of diameter a (Ref. 1). The differences are minimal in the symmetric situation considered and are also shown in Fig. 2. It is seen that for $a < 4.0$, the best approximation is that of equal volumes of square and circular cylinders. For $a > 4.0$, the first-collision probability of a square cylinder should be approximated by squaring the slab probability.

*N. J. McCormick**

Department of Nuclear Engineering
The University of Michigan
Ann Arbor, Michigan

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*AEC Predoctoral Fellow

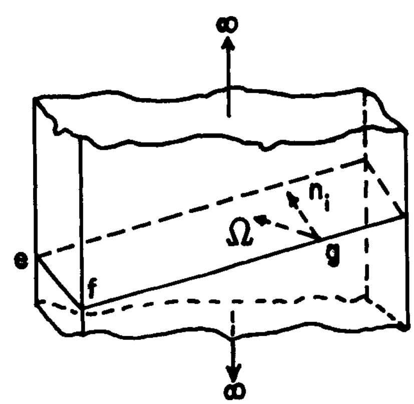

Fig. 1. Cylinder Geometry

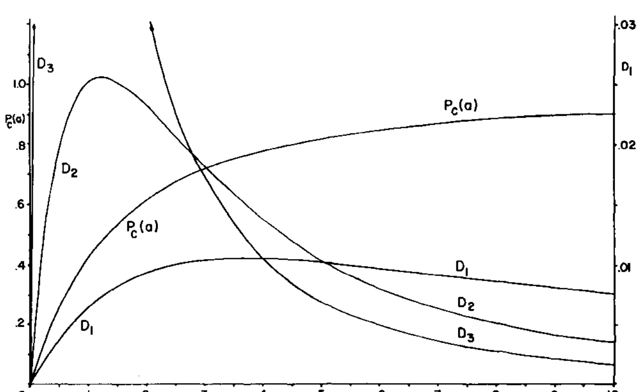

Fig. 2. First-Collision Probability and Probability Differences for Square Cylinder

[1] K. M. CASE, F. de HOFFMANN, and G. PLACZEK, *Introduction to the Theory of Neutron Diffusion*, Vol. 1, U. S. Govt. Printing Office, Washington 25, D. C., 1953.

[2] L. DRESNER, *Nucl. Sci. Eng.* 6, 63 (1959).