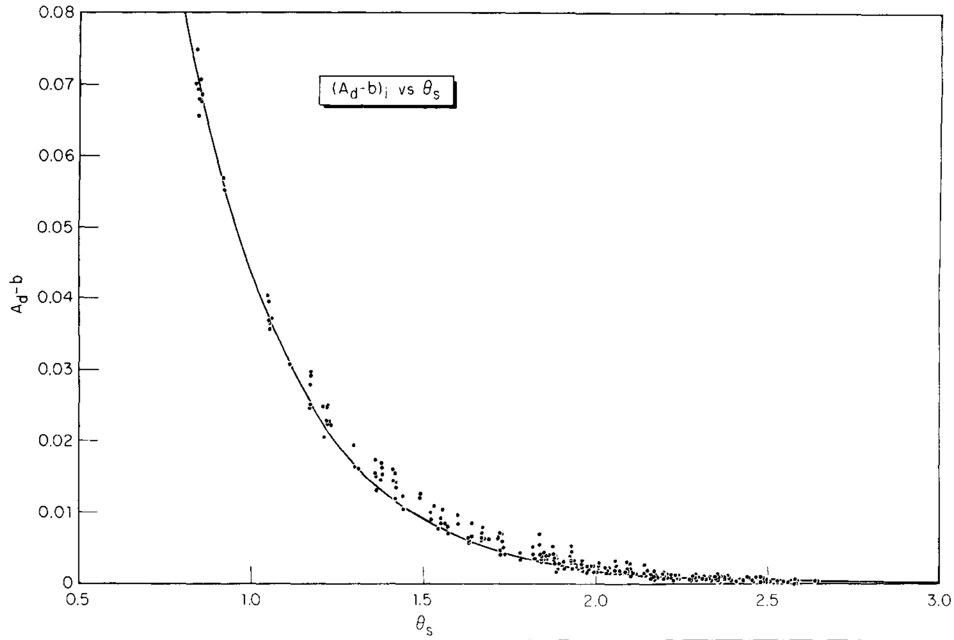

Cos θ_0	Source	Scatterer	b
0.500	Co^{60}	Fe	0.0070
√	√	Al	0.0085
√	√	Concrete	0.0085
√	Cs^{137}	Fe	0.0091
√	√	Al	0.0132
√	√	Concrete	0.0133
0.750	Co^{60}	Fe	0.0080
√	√	Al	0.0113
√	√	Concrete	0.0107
√	Cs^{137}	Fe	0.0120
√	√	Al	0.0167
√	√	Concrete	0.0165
1.000	Co^{60}	Fe	0.0078
√	√	Al	0.0114
√	√	Concrete	0.0117
√	Cs^{137}	Fe	0.0130
√	√	Al	0.0194
√	√	Concrete	0.0184

Fig. 1. The quantity A_d-b (see text) plotted versus Compton scattering angle θ_s, in radians. The points represent experimental data taken for the various conditions listed on the figure, and the smooth line is the function $e^{-\pi\theta_s}$.

A USNRDL technical report is in preparation which gives the full details of the above note as well as a description of the dose albedo experiment, a tabulation of its results, and a comparison with the Raso[1] results as modified by the Chilton-Huddleston[2] semi-empirical formula.

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A Note on the Analysis of Pulsed Neutron Shutdown Measurements

The reactivity of a subcritical reactor is frequently determined by injecting bursts of fast neutrons and measuring the decay constant of the fundamental flux mode (α_1). The reactivity is related to α_1 by the expression:

$$\frac{\rho_1}{\bar{\beta}_1} = -\left(\alpha_1 - \frac{\bar{\beta}_1}{\Lambda_1}\right)\Big/\frac{\bar{\beta}_1}{\Lambda_1}\ , \qquad (1)$$

where $\bar{\beta}_1$ is the effective delayed-neutron fraction and Λ_1 is the generation time. Garelis and Russell[1] have shown that the term $\bar{\beta}_1/\Lambda_1$ (or $k\bar{\beta}_1/\ell_1$) can be determined from the shape of the measured pulse using the expression:

$$\int_0^\infty N_p \cdot \exp\left(\frac{\bar{\beta}_1}{\Lambda_1} \cdot \tau\right) d\tau - \int_0^\infty N_p \cdot d\tau = \frac{N_d}{R}, \qquad (2)$$

where N_p is the prompt contribution to the neutron density
N_d is the effectively constant 'delayed' neutron contribution
τ is the time after a pulse
R is the repetition rate.

The analysis leading to this expression[1] was limited to a bare slab core with only one neutron energy group; this last restriction probably is the most severe since it leads to very simplified expressions for the multiplication constant and lifetime. The alternative analysis outlined below provides an approximate treatment of the energy variable and defines the terms involved. It is

[1]E. GARELIS and J. L. RUSSELL, Jr., *Nucl. Sci. Eng.*, **16**, 263-270 (1963).

shown that Eq. (2) is strictly correct for only the fundamental spatial mode and for moderate shutdowns, although it may be sufficiently accurate for many applications.

The time-dependent diffusion equation for a uniform slab reactor is:

$$D(u)\frac{\partial^2 F}{\partial x^2} - \Sigma(u)F + \int_0^\infty du'\left[\Sigma_s(u'\to u) + \frac{1}{\lambda_0}f(u)\nu(u')\Sigma_f(u')(1-\beta)\right]F(x,u',t) + S(x,u,t) + \sum_i \lambda_i C_i(x,t)f_i(u) = \frac{\partial}{\partial t}\left[\frac{F}{v(u)}\right], \tag{3}$$

where

$$\frac{\partial C_i(x,t)}{\partial t} = \frac{1}{\lambda_0}\int_0^\infty du'\cdot\nu(u')\Sigma_f(u')\beta_i F(x,u',t) - \lambda_i C_i(x,t)\,. \tag{4}$$

Here $F = F(x,u,t)$ is the neutron flux at point x and time t for neutrons of lethargy u

$\Sigma(u)$ is the total macroscopic cross section for absorption and scattering.

$\Sigma_s(u'\to u)$ is the macroscopic cross section for scattering from lethargy u' to lethargy u

Σ_f is the fission cross section

$\nu(u)$ is the number of neutrons emitted in a fission process induced by neutrons of lethargy u

$f(u)$ and $f_i(u)$ are the energy spectra of prompt and i-th group delayed neutrons respectively, normalised to unity for the integral over all lethargy

$C_i(x,t)$ and λ_i are the concentration and decay constant for the i-th group of delayed neutron precursors

β_i is the delayed neutron fraction for group i and $\beta = \sum_i \beta_i$

$v(u)$ is the speed of neutrons of lethargy u

λ_0 is an eigenvalue, equal to unity in this case, but introduced for consistency with Eq. (7) below. λ_0 can also be used to allow for any bias in the constants as determined from critical size calculations.

If the source is considered to be a series of delta functions in time injected at a rate of R/sec, then between the pulses the neutron flux and precursor concentrations satisfy Eqs. (3) and (4) with $S(x,u,t)$ set equal to zero. $F(x,u,t)$ and $C_i(x,t)$ can be expanded in terms of the eigenfunctions of the bare slab core, $\psi_n(x)$, i.e.

$$F(x,u,t) = \sum_{n=1}^{\infty} \Phi_n(u,t)\cdot\psi_n(x)$$

and

$$C_i(x,t) = \sum_{n=1}^{\infty} C_{in}(t)\cdot\psi_n(x)\,,$$

and the equations for the n-th mode coefficients are now:

$$-D(u)B_n^2\Phi_n(u,t) - \Sigma(u)\Phi_n(u,t) + \int_0^\infty du'\left[\Sigma_s(u'\to u) + \frac{1}{\lambda_0}f(u)\nu(u')\Sigma_f(u')(1-\beta)\right]\times \Phi_n(u',t) + \sum_i \lambda_i C_{in}(t)f_i(u) = \frac{\partial}{\partial t}\left[\frac{\Phi_n(u,t)}{v(u)}\right] \tag{5}$$

$$\frac{dC_{in}(t)}{dt} = \frac{1}{\lambda_0}\int_0^\infty du'\cdot\nu(u')\Sigma_f(u')\beta_i\Phi_n(u',t) - \lambda_i C_{in}(t) \tag{6}$$

where B_n^2 is the n-th mode geometric buckling defined in the usual manner.

If only a single neutron pulse is considered, the neutron spectrum of each spatial mode will vary with time until it reaches an equilibrium spectrum in a time of the order of the greatest delayed neutron half-life (i.e., ~ 56 sec). However, if the reactor is pulsed continuously for a time long compared to 56 secs, with a repetition rate, R, much greater than the largest λ_i (i.e., $R \gg 3.00$ sec^{-1}), then the neutron flux will consist of an approximately constant 'delayed' neutron contribution and a time-dependent prompt neutron contribution. The energy spectrum of the prompt contribution to each mode will now reach the equilibrium spectrum in a time of the order of the generation time (Λ) after a pulse, since after this time all the neutrons present will be the products of core fissions. The spectrum of the 'delayed' tail will also be approximately the same as that of the prompt contribution provided the reactor is not too shutdown, i.e., $|-\rho| < 15\%\ \delta k/k$. This is because the delayed tail consists of not only delayed fission neutrons but also their progeny, and the latter will predominate if there is appreciable multiplication. Consequently, after about a generation time, the n-th mode flux will be separable into functions of lethargy and time, i.e.

$$\Phi_n(u,t) = \phi_n(u)\cdot T_n(t)\,.$$

Equation (2) is deduced below assuming that this separation holds for all values of the time. This assumption is discussed further at the end of this letter.

The source-free stationary adjoint function for each mode $\phi_n^*(u)$ will now be defined for a hypothetical reactor of the same composition as the actual reactor, but with the eigenvalue λ_n adjusted to give a stationary situation. The adjoint is defined by:

$$-D(u)B_n^2\phi_n^*(u) - \Sigma(u)\phi_n^*(u) + \int_0^\infty du'\cdot\left[\Sigma_s(u\rightarrow u') + \frac{1}{\lambda_n}\left(f(u')(1-\beta)+\sum_i f_i(u')\beta_i\right)\nu(u)\Sigma_f(u)\right]\phi_n^*(u')=0. \tag{7}$$

This equation is the same as the diffusion equation for $\phi_n(u)$ except that u and u' have been interchanged in the source terms in the square brackets.

Following the method of analysis given by Henry[2], multiply Eq. (5) by $\phi_n^*(u)$ and integrate over u and then subtract Eq. (7) first multiplied by $\phi_n(u)\cdot T_n(t)$ and then integrated over u. This leads to:

$$\frac{dT_n}{dt} = \frac{\rho_n - \bar{\beta}_n}{\Lambda_n}\cdot T_n + \sum_i \lambda_i c_{in}, \tag{8}$$

where $\bar{\beta}_n = \sum_i \bar{\beta}_{in}$ is the effective delayed-neutron fraction for the n-th mode and $\bar{\beta}_{in}$ is given by:

$$\bar{\beta}_{in} = \frac{1}{\lambda_0}\cdot\frac{\beta_i}{G_n}\cdot\int_0^\infty du\cdot f_i(u)\phi_n^*(u)\cdot\int_0^\infty du'\cdot\nu(u')\times \Sigma_f(u')\phi_n(u'), \tag{9}$$

where

$$G_n = \int_u\int_{u'}\left[f(u)(1-\beta)+\sum_i f_i(u)\beta_i\right]\times \nu(u')\Sigma_f(u')\phi_n(u')\phi_n^*(u)\cdot du\cdot du'; \tag{10}$$

c_{in} is the effective precursor concentration given by

$$c_{in} = \frac{1}{\Lambda_n G_n}\cdot\int_u du\cdot C_{in}(t)f_i(u)\phi_n^*(u)$$

and Λ_n is the n-th mode generation time, defined by

$$\Lambda_n = \frac{1}{G_n}\int_u du\cdot\frac{\phi_n(u)\phi_n^*(u)}{v(u)}. \tag{11}$$

[2]A. F. HENRY, *Nucl. Sci. Eng.*, **3**, 52-70 (1958).

It should be noted that the definition of G_n is arbitrary, and has been chosen so that the n-th mode reactivity is now defined by:

$$\rho_n = \frac{\lambda_n - \lambda_0}{\lambda_n\lambda_0} = \frac{\lambda_n - 1}{\lambda_n} \quad \text{if} \quad \lambda_0 = 1$$

which in the case of the fundamental is the conventional definition of reactivity. The λ_1 used here is the eigenvalue that would be obtained from a solution of the stationary diffusion equation. Multiplying Eq. (6) by $\phi_n^*(u)f_i(u)$ and integrating over u gives:

$$\frac{dc_{in}}{dt} = \frac{\bar{\beta}_{in}}{\Lambda_n}\cdot T_n - \lambda_i c_{in}. \tag{12}$$

Equations (8) and (12) have the same form as the conventional point kinetics equations.

The numerical values of Λ_n and $\bar{\beta}_{in}$ can be calculated by perturbing the hypothetical stationary reactor. Λ_n is equal to the reactivity change $\Delta\rho$ produced by adding a negative unit $1/v$ absorber, and $\bar{\beta}_{in}$ is equal to the reactivity change produced by adding $\lambda_0 f_i(u)\beta_i\nu(u')$ to the source of neutrons of lethargy u produced per fission, i.e.

$$\delta\left\{\frac{1}{\lambda_n}\left[f(u)(1-\beta)+\sum_i f_i(u)\beta_i\right]\nu(u')\right\} = \lambda_0 f_i(u)\beta_i\nu(u'),$$

where λ_n is the eigenvalue of the unperturbed hypothetical stationary reactor for the n-th mode.

Since it is assumed that $R \gg \lambda_i$, the delayed neutron precursor concentrations will be approximately independent of time, and the solution of Eq. (8) is

$$T_n = A_n\exp(-\alpha_n\tau) + E_n, \tag{13}$$

where

$$\alpha_n = \frac{\bar{\beta}_n - \rho_n}{\Lambda_n} \tag{14}$$

and

$$E_n = \frac{1}{\alpha_n}\cdot\sum_i \lambda_i c_{in}. \tag{15}$$

τ is the time after a pulse at time $\tau = 0$.

Providing $R \ll \alpha_1$ the prompt contributions from different pulses do not overlap and the solution need be carried out for only one pulse, the contribution from M pulses being M times the value given here.

Equation (14) shows that for the fundamental mode:

$$\frac{\rho_1}{\bar{\beta}_1} = -\left(\alpha_1 - \frac{\bar{\beta}_1}{\Lambda_1}\right)\Big/\frac{\bar{\beta}_1}{\Lambda_1}, \qquad \text{which is Eq. (1).}$$

Separating T_n into prompt (T_{pn}) and delayed (T_{dn}) contributions gives

$$T_{pn} = A_n \exp(-\alpha_n \tau) \tag{16}$$

and

$$T_{dn} = E_n = \frac{1}{\alpha_n} \cdot \sum_i \lambda_i c_{in} \, . \tag{17}$$

The expression for T_{dn} can be simplified by using Eq. (12) which shows that:

$$T_{dn} = \frac{1}{\alpha_n} \cdot \sum_i \lambda_i c_{in} = \frac{\bar{\beta}_n}{\Lambda_n} \cdot \frac{\bar{T}_n}{\alpha_n} \, , \tag{18}$$

where $\bar{T}_n$ is the average value of T_n given by:

$$\bar{T}_n = R \int_0^{1/R} T_n(\tau) d\tau = \frac{RA_n}{\alpha_n} \left[1 - \exp\left(-\frac{\alpha_n}{R}\right)\right] + T_{dn} \, .$$

Consequently, if $R \ll \alpha_n$ for all n, the exponential can be set equal to zero, giving

$$\bar{T}_n = \frac{RA_n}{\alpha_n} + T_{dn} \, . \tag{19}$$

Inserting this value into Eq. (18) and solving for T_{dn} gives:

$$T_{dn} = \frac{RA_n \cdot \bar{\beta}_n/\Lambda_n}{\alpha_n(\alpha_n - \bar{\beta}_n/\Lambda_n)} \, . \tag{20}$$

Following the procedure of Ref. 1, define the integrals I_n^1 and I_n^2 by:

$$I_n^1 = \int_0^{1/R} T_{pn} \, d\tau \simeq \int_0^{\infty} T_{pn} \, d\tau \quad \text{since } R \ll \alpha_n$$

$$I_n^2 = \int_0^{1/R} T_{pn} \cdot \exp\left(\frac{\bar{\beta}_1}{\Lambda_1}\tau\right) d\tau \simeq \int_0^{\infty} T_{pn} \cdot \exp\left(\frac{\bar{\beta}_1}{\Lambda_1} \cdot \tau\right) d\tau .$$

Then using Eq. (16) for T_{pn},

$$I_n^2 - I_n^1 = \frac{A_n \cdot \bar{\beta}_1/\Lambda_1}{\alpha_n(\alpha_n - \bar{\beta}_1/\Lambda_1)} \, . \tag{21}$$

It can be seen that $I_n^2 - I_n^1$ equals T_{dn}/R when $n = 1$. If it is assumed for the moment that $\bar{\beta}_n/\Lambda_n$ equals $\bar{\beta}_1/\Lambda_1$, then

$$I_n^2 - I_n^1 = \frac{T_{dn}}{R} \tag{22}$$

for all modes.

If Eq. (22) is multiplied by $\phi_n(u) \cdot \psi_n(x)$ and by the interaction cross section of an in-core neutron detector, then integration over lethargy, and summation over all modes, gives:

$$\int_0^{\infty} N_p \cdot \exp\left(\frac{\bar{\beta}_1}{\Lambda_1} \cdot \tau\right) d\tau - \int_0^{\infty} N_p d\tau = \frac{N_d}{R} \, , \tag{23}$$

which is the relation given by Garelis and Russell[1].

Equation (23) has been derived for an idealized pulse shape in which it is assumed that the flux is separable into functions of lethargy and time. As discussed above, this is not true for times of the order of the generation-time after a pulse, and it is difficult to assess the error. However, when the results of an actual experiment are inserted into Eq. (23) the effect on the left-hand side of the equation is negligible since the integrals cancel for small values of τ. It can be shown using Eq. (16), and considering only the fundamental mode, that the contribution to the left-hand side for times less than Λ_1 is expected to be only a fraction $\sim \frac{1}{2}\rho_1^2$ of the total contribution. It can also be shown by a similar method that only a fraction $\sim \rho_1$ of the total background would be expected to be attributable to fissions occurring at times less than Λ_1 after a pulse. Consequently, the actual pulse shape can differ significantly from the assumed pulse shape for times less than Λ_1 without causing an appreciable error, at least for negative reactivities up to about 15% $\delta k/k$.

A second assumption made in deriving Eq. (23) was that $\bar{\beta}_n/\Lambda_n$ is independent of mode number n. This ratio is given by Eqs. (9) and (11) as:

$$\frac{\bar{\beta}_n}{\Lambda_n} = \frac{\frac{1}{\lambda_0}\int_{u'} du' \cdot \nu(u')\Sigma_f(u')\phi_n(u') \cdot \left[\sum_i \beta_i \int_u du \cdot f_i(u)\phi_n^*(u)\right]}{\int_u du \cdot \frac{\phi_n(u)\,\phi_n^*(u)}{v(u)}}$$

and will change with mode because of the different leakage terms in both the flux and adjoint equations. For a very large reactor, the ratio may not change greatly for the first few harmonics, while in the case of a small reactor the higher harmonics usually decay so rapidly that they do not appreciably affect the terms of Eq. (23) provided the source and detector locations are suitably chosen. However, it is necessary to examine experimental results to verify that Eq. (23) is valid. A similar conclusion can be reached from the analysis of Ref. 1 if $\bar{\beta}$ is assumed to be a function of mode.

The analysis presented above shows that pulsed neutron shutdown measurements made in cores that can be considered as effectively uniform and unreflected can be analyzed by the Garelis and Russell method, provided the terms of Eq. (23) are not appreciably affected by spatial harmonics. It is also shown that the deduced negative-reactivity values can be directly related to the eigenvalue of the stationary diffusion equation.

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