fission products with a scintillation counter[1,2]; another uses a fission counter to detect fission fragments directly[3]. The latter technique is usually difficult to use because of the physical size of the fission counter. Recently, Bigham[3] suggested that in the gamma-counting technique, differences in thermal and epithermal fission yields would not show up as differences in decay rate if these fission products were short-lived or detected with only low efficiency. To investigate this problem, an experimental study of the U^{235} and U^{233} fission cadmium ratios has been carried out, using both of the methods outlined above.

The experiment was performed using pairs of highly enriched U^{235} (or U^{233}) deposits in a double chamber back-to-back fission counter[4]. Highly enriched U^{235} (or U^{233}) foils were sandwiched between the deposits. This type of counter was used to ensure that the effect due to any flux gradients would be eliminated. Subsequent analysis of the data indicated that this precaution was not necessary. The deposits were 0.1-mg/cm^2 and 0.01-mg/cm^2 thick for U^{235} and U^{233}, respectively, yielding plateaus having a slope of 0.2%/V over a 50-V range. The 0.005-in. (0.013 cm) foils were made from a 2.5 wt% uranium-aluminum alloy enriched to greater than 90% U^{235} (or U^{233}). The self-shielding of these foils was less than 1%.

The irradiation (30 min) was done with the fission counter bare, and repeated with the counter cadmium-covered. The same deposits and foils were used in each set of irradiations to eliminate the need for foil and deposit intercalibrations. Special precautions were taken to ensure that the counter was completely covered to eliminate thermal-neutron streaming paths. This was necessary since the cadmium ratio was approximately 40. The cadmium ratios were determined in two ways for each set of bare and cadmium-covered irradiations. In one, the total fissions from the deposits were recorded during the irradiation; in the other, after the irradiation the fission-product activities of the foils were counted on a 2-in. (5.1 cm) NaI scintillation counter biased to reject pulses below 400 keV. The U^{235} foils were counted between 50 to 100 min after the end of the irradiation, and the U^{233} foils were counted in the interval 30 to 90 min after irradiation. These time intervals are similar to those used in resonance integral determinations[1]. A Cs^{137} standard was used to monitor any drift in the scintillation counter.

Eight sets of cadmium-ratio determinations were obtained for U^{235} and four sets for U^{233}. Corrections were made for counting losses (less than 0.7%), foil self-shielding (less than 0.6%), and for power level drift during the irradiation (less than 1%).

The ratio of the cadmium ratios determined by fission counting to those determined by gamma counting yielded a value of 1.005 ± 0.007 for U^{235} and a value of 1.002 ± 0.005 for U^{233}. These values support the use of the fission-product gamma-counting method in the measurement of U^{235} and U^{233} cadmium ratios, and also establish the validity of this technique as applied to determinations of these fissile infinite dilution resonance integrals.

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Intermediate Resonance Absorption at Low Energies

Is moderator scattering adequately treated in a narrow resonance (*NR*) approximation for low-energy resonances? For cases in which light moderating elements are admixed with the fuel, this may be a particularly important question. Recent work[1,2,3] indicates that the *NR* approximation may not always be applicable to the moderator. The first two references treat this problem for resonance absorption in nonhomogeneous systems; the third concentrates on an examination of the flux shape.

We consider a simple extension of the 'intermediate resonance' (*IR*) formulation[4] of the problem for homogeneous systems. The extension to nonhomogeneous systems may be made by means of equivalence principles[5].

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Consider the basic equation for a spatially independent homogeneous mixture of a nonabsorbing moderator and a resonance absorber:

$$(\sigma_m + \sigma)\psi(u) = K_m \sigma_m \psi + K\sigma_s \psi \,. \tag{1}$$

The notation is consistent with that of Ref. 4, except that σ_m is the moderator scattering per absorber atom. The flux is normalized to unity far above the resonance; $\psi(u)\xrightarrow[u\to-\infty]{}1$. K is the slowing-down integral operator for the absorber,

$$K = \int_{u-\Delta}^{u} du' \frac{e^{-(u-u')}}{1-\alpha} \,, \quad \Delta = \ln\frac{1}{\alpha} \,, \tag{2}$$

and K_m is the corresponding operator for the moderator. In Ref. 4 the term $K_m\sigma_m\psi$ was replaced by σ_m, the result of applying the *NR* approximation to the moderator. This restriction is now being removed.

A first-order solution to Eq. (1) may be written as

$$\psi^{(1)}_{\kappa\lambda} = \frac{\kappa\sigma_m + \lambda\sigma_p}{\kappa\sigma_m + \sigma_a + \lambda\sigma_s} = \frac{1 + x^2}{\beta^2_{\kappa\lambda} + x^2} \,, \tag{3}$$

where

$$\beta^2_{\kappa\lambda} = 1 + \frac{\sigma_0}{\kappa\sigma_m + \lambda\sigma_p} \quad \frac{\Gamma_\gamma + \lambda\Gamma_n}{\Gamma} \,. \tag{4}$$

Here κ is the *IR* parameter for the moderator and λ is the corresponding quantity for the absorber. Thus, for example, with $(\kappa,\lambda) = (1,0)$, Eq. (3) gives the first-order *NR-WR* (or *NRIA*) solution to Eq. (1). This means that the *NR* approximation has been applied to the moderator, while the wide resonance (*WR*) approximation has been applied to the absorber.

As in Ref. 4, first-order resonance integrals are given directly in terms of the quantity $\beta_{\kappa\lambda}$:

$$I^{(1)}_{\kappa\lambda} = \frac{I^{(0)}}{\beta_{\kappa\lambda}} \,. \tag{5}$$

By setting κ or λ equal to zero or unity, one obtains various combinations of first-order results.

In the intermediate resonance approach, we calculate second-order resonance integrals by iterating the first-order solution (3) in the basic equation (1), and then equate successive orders of approximation as a means of solving for the parameters κ and λ. In evaluating the second-order resonance integrals, we also make use of the approximate form of the slowing-down operator,

$$K = \frac{1}{\delta}\int_{x}^{x+\delta} dx' \,; \qquad \delta = 2E_r(1-\alpha)/\Gamma \,. \tag{6}$$

Note that this approximation to K, which enables us to get simple analytical results, also preserves the property of the exact K, that when it operates on a constant it gives back the constant ($K \cdot 1 = 1$).

Setting $I^{(2)}_{\kappa\lambda} = I^{(1)}_{\kappa\lambda}$ yields the following relation:

$$\sigma_m(\Gamma_\gamma + \lambda\Gamma_n)[1 - X^{(m)}_{\kappa\lambda} - \kappa] = (\sigma_m\kappa\Gamma_n - \sigma_p\Gamma_\gamma)[1 - X_{\kappa\lambda} - \lambda] \,, \tag{7}$$

where

$$X_{\kappa\lambda} = \frac{1}{x_{\kappa\lambda}}\tan^{-1}x_{\kappa\lambda} \,, \qquad x_{\kappa\lambda} = \frac{\delta}{\beta_{11} + \beta_{\kappa\lambda}} \,, \tag{8}$$

and $X^{(m)}_{\kappa\lambda}$ is the same as $X_{\kappa\lambda}$ except that δ is replaced by

$$\delta_m = \frac{2E_r(1-\alpha_m)}{\Gamma} \,, \quad \text{for the moderator.}$$

Since Eq. (7) involves two parameters, an additional condition is necessary in order to obtain an an explicit solution. However, to keep our results simple, we choose to make some physical arguments and use some results of Ref. 4. In the *NR* limit with respect to the absorber, $x_{\kappa\lambda} >> 1$ and the absorber parameter λ goes to unity. In this limit, therefore, the right-hand side of Eq. (7) vanishes, and hence, for this equation to be satisfied we require that $1 - X^{(m)}_{\kappa1} - \kappa = 0$. Similarly, in the absorber *WR* limit, $x_{\kappa\lambda} << 1$ and $\lambda \to 0$. This means that for Eq. (7) to be satisfied, $1 - X^{(m)}_{\kappa0} - \kappa = 0$.

We stipulate, now, for all values of λ, that the moderator square bracket vanish:

$$1 - X^{(m)}_{\kappa\lambda} - \kappa = 0 \,. \tag{9a}$$

But for Eq. (7) to be satisfied for all λ, we also need

$$1 - X_{\kappa\lambda} - \lambda = 0 \,. \tag{9b}$$

The pair of coupled transcendental equations (9a,b) form an explicit solution for the parameters κ and λ in terms of the resonance parameters. The equations may be solved numerically by iteration. When the solutions for κ and λ are inserted into Eq. (5), one has the *IR* approximation to the resonance integral.

For example, consider the 6.68 eV resonance of U^{238} for a 1:1 atomic mixture of hydrogen and uranium ($\sigma_m = 20.2$, $\sigma_p = 10$). Starting with $(\kappa,\lambda) = (1,0)$, which is the standard *NR-WR* or *NRIA* approximation for this resonance, one obtains after only two iterations $(\kappa,\lambda) = (0.821, 0.0062)$ as the solution to Eqs. (9). The values of the parameters indicate that the absorber *WR* approximation is more applicable than the moderator *NR* approximation. The *IR* integral for this case is 3.65 b. A numerical integration of the equations gives 3.76 b, so that in comparison, the *IR* result has a 2.9% error. On the other hand, the *NR-WR* result is 4.02 b, which has a 6.9% error.

For heavier moderators the errors are even more significant. For a carbon-uranium mixture such that σ_m = 20.2, the solution to Eqs. (9) for this resonance is $(\kappa,\lambda) = (0.427, 0.0042)$. Here the moderator *NR* approximation is clearly no longer applicable and the *NR-WR* approximation has a 56% error, whereas the *IR* result agrees to within 1.9% of the numerical result. These results are summarized in Table I.

TABLE I

Resonance Integrals (in barns) for the 6.68 eV Resonance of U^{238} ($\sigma_m + \sigma_p = 30.2$)

Moderator	*NR-WR*	*IR*	Numerical
Hydrogen	4.02	3.65	3.76
Carbon	4.02	2.63	2.58

It is clear, therefore, that the *NR* approximation is not always applicable to the moderator. It is particularly bad for the lower energy resonances and the heavier moderating elements. For the higher energy resonances the error of the moderator *NR* approximation is less significant. Furthermore, as the moderator concentration increases, the error decreases because the flux depression in the resonance decreases and the resonance integral becomes less sensitive to the various approximations. In all cases, however, it is not difficult to solve the appropriate *IR* equations (9a,b) and obtain the corresponding resonance integral (5) for any resonance and any mixture.

The extension to nonhomogeneous systems may be made by representing the lump by an effective scattering cross section s. As in Ref. 5, s is given in terms of the lattice characteristics C and $\bar{\ell}$ by

$$s = \frac{1 - C}{N\bar{\ell}} . \qquad (10)$$

All of the results in this paper now apply, with the only change being that $\beta_{\kappa\lambda}$ is given by

$$\beta^2_{\kappa\lambda} = 1 + \frac{\sigma_0}{s + \kappa\sigma_m + \lambda\sigma_p} \; \frac{\Gamma_\gamma + \lambda\Gamma_n}{\Gamma} . \qquad (11)$$

The σ_m in Eq. (11) now applies to any moderator which may be admixed with the fuel.

We have used the equivalence expressed by Eq. (10) in order to keep our results simple, but its use for nonhomogeneous systems introduces additional approximations. In particular, the moderator outside the lump is necessarily treated in an *NR* approximation. An appropriate example would be UO_2 fuel rods in water. Here, σ_m would be the scattering cross section of the oxygen, treated in an *IR* approximation, while s represents the water scattering, as treated in an *NR* approximation.

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On Transfer Cross Sections Between Overlapping Thermal Groups

In a system of moderators at different temperatures, neutrons which have been thermalized in one medium may diffuse into another, where they may suffer collisions and eventually enter the energy-distribution characteristic of this medium. The rate of transfer of neutrons from the first distribution into the second is characterized by the so-called 'transfer' or 'rethermalization' cross sections. This process of rethermalization has been investigated by several authors[1-7] in different ways.

The present note intends to demonstrate the connection between the results of the mentioned authors, by investigating concisely the case of two purely scattering moderators at different temperatures (Kottwitz's problem).

We assume that the thermal flux Φ in the system can be approximated by a superposition of two appropriate energy distributions $\phi_i(E)$ $(i = 1,2)$ weighted by spatially dependent factors:

$$\Phi(\boldsymbol{r},E) = \sum_{i=1}^{2} F_i(\boldsymbol{r}) \cdot \phi_i(E) . \qquad (1)$$

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