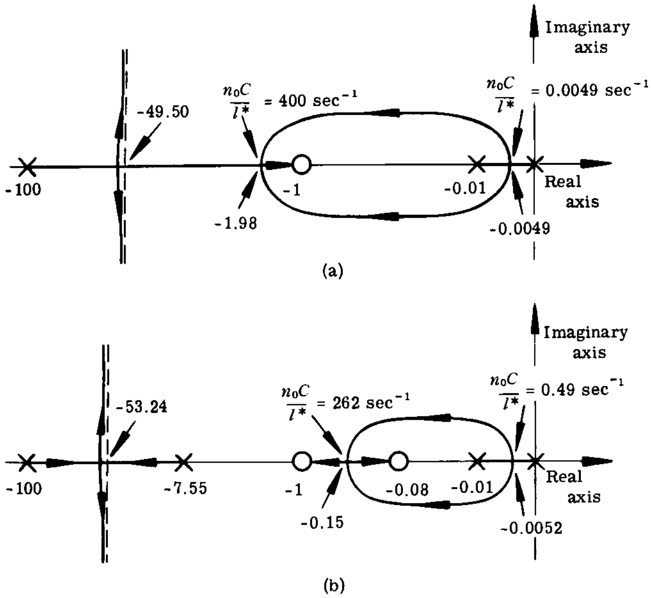


Fig. 5. Root-locus diagrams.

(a) A sketch of the root-loci for

$$1 + \frac{n_0C}{l^*}\frac{(s+1)}{s\,(s+0.01)\,(s+100)} = 0$$

(b) A sketch of the root-loci for

$$1 + \frac{n_0C}{l^*}\frac{(s+1)\,(s+0.08)}{s\,(s+0.01)\,(s+100)\,(s+7.55)} = 0\ .$$

system, this would reveal the stabilizing rather than the destabilizing effect of delayed neutrons.

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## "Reply to 'On the Stabilizing Effect of Delayed Neutrons' "

In the above Letter to the Editor, Professor Tan shows that the so-called "stabilizing effect of delayed neutrons" can be considered from vastly different angles, and she studies the variation of the *degree of stability when the power level is increased.* While this analysis is basically correct, it would be wrong to believe that it contradicts the conclusions given in Ref. 1, because in this reference the interest lay only in *comparing the dynamic properties* of reactors with and without delayed neutrons *at the same power level,* knowing that reactors without delayed neutrons did not exist and that, therefore, the comparison was fairly academic.

More precisely, in relation to Fig. 2a, above, Professor Tan agrees with the single conclusion regarding conditional stability reactors given in Ref. 1. The ensuing discussion by Professor Tan on the degree of stability simply completes the analysis given previously where this aspect was not considered. To conclude, as above, that this "example is not acceptable as an evidence of the destabilizing effects of delayed neutrons" seems, therefore, illogical because the reactor model does become unstable when the parameter $\beta$ is increased from zero to its real, non-zero, physical value.

In Ref. 1, two examples of reactors in which delayed neutrons enhance the oscillatory behavior of the transient response are given. With the feedback kernel given erroneously in the caption of Fig. 6, it is true that the enhancement of the oscillations cannot be shown. However, it is easily seen that for reactors having feedbacks similar to the one drawn up in Fig. 6 of Ref. 1, the effect of delayed neutrons can be to make the response more oscillatory, (cf., curves A and B of Fig. 2b, above).

A second example is dealt with in Ref. 1, and it is concluded that "delayed neutrons enhance the oscillatory behavior of the solutions." To write that Smets has come to the conclusion that the reactor without delayed neutrons is more stable than with delayed neutrons is groundless, because it was not written and because the question of the degree of stability was not even considered.

This example can also be examined in terms of the root locus, provided that one does not assume, as done above, that $\omega$ is given in terms of rad/sec. The high frequency part of Figs. 7 and 8 (Ref. 1) shows that $\lambda > 100$ and, therefore, that the zero at $-\lambda$ and the pole as $-(\beta/l)$ of Fig. 5b above are at the left of the pole -100. Hence, one should compare the root loci of

$$1 + \frac{n_0C}{l}\frac{(s+1)}{s(s+0.01)(s+100)} = 0$$

and

$$1 + \frac{n_0C}{l}\frac{(s+1)(s+\lambda)}{s(s+0.01)(s+100)(s+\beta/l)} = 0\ .$$

In the region of the complex plane $s \ll 100$, $s$ can be ignored with regard to $\lambda$ and $\beta/l$. The root locus is not altered but the gain is reduced. If $n_0C/l \simeq 400$, the transient response for $\beta = 0$ is aperiodic (critical damping, double root near -2) and when $\beta > 0\ \{[(\lambda l)/\beta] = 10^{-2}\}$ the dominant roots are $s \simeq -0.025 \pm j\ 0.2$. Therefore, the reactor with delayed neutrons has a weakly damped *oscillatory* response while the reactor without delayed neutrons has a strongly damped aperiodic response.

In conclusion, Professor Tan's analysis precisely shows conditions under which delayed neutrons may make the reactor more stable (or less stable). It does not disprove the statements made previously:

1. A reactor can be unstable, when $\beta > 0$ and asymptotically stable in the small, when $\beta = 0$.
2. In some reactors, the effect of the delayed neutrons is to enhance the oscillatory behavior of the transient response.
3. Consideration of the peaks in the closed-loop transfer function provides information on the transient response (in particular the location of the dominant roots in the complex plane).

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[1]H. B. SMETS, *Nucl. Sci. Eng.*, **25**, 236 (1966).