Letters to the Editor

Comment on 'A Space-Dependent Reactor-Noise Formulation Utilizing Modal Expansions'

In a recent paper by Danofsky[1] "Sink frequencies" are exhibited for both auto- and cross-spectral-density functions. In the case of the auto-spectral-density function the sink frequency is apparently caused by the presence of a local stochastic absorber which dominates all other noise sources. The sink frequencies associated with cross-spectral densities are caused by space-dependent observation points when the noise sources are spatially distributed. The paper fails to distinguish, however, between important differences in the character of the auto and cross spectra in the vicinity of the sink frequency. The purpose of this note is to point out these differences.

In the first place, auto-spectral densities are real and non-negative. For this reason, the auto spectra at frequencies both above and below the sink frequency are positive. The value of the spectrum at the sink frequency may be zero or any positive value. Unfortunately, the value of the spectrum at the sink frequency is not given.

In contrast to the auto-spectrum, the cross-spectral density is, in general, a complex function. However, only the magnitude of the cross spectrum is shown in the curves given in the paper. If one decomposes the cross-spectral density into its real and imaginary parts, it becomes apparent that the sink frequency effect may have quite a different interpretation. For example, it has been shown[2] that in the special case of symmetrically located observation points the cross spectrum is always real. Furthermore, if only simple two-node space dependence is assumed then the sink frequency of the cross spectrum is a null frequency,[3,4] that is, the cross spectrum is zero at the sink frequency. Also, in this case, the cross spectrum is positive for frequencies below the null (or sink) frequency and negative for frequencies above the null frequency. A near-symmetrical case is the case of the cross spectrum between space points 1 and 4 in the paper. It would be interesting to know the amplitude of the imaginary part of the spectrum in this case, as well as the sign of the real part. It would also be interesting to know the real and imaginary parts of the cross spectra in the asymmetrical cases reported.

The paper made no effort to correlate the value of the sink frequency with the parameters of the reactor. It has been found previously[3,4] that the degree of decoupling of the two fuel regions and the mean time delay for the propagation of a disturbance from one region to the other determine the position of the sink frequency. There must be some analogous relationship when a modal model is used which relates the properties of higher modes to the value of the sink frequency.

It has been shown previously,[5] that the *coherence function* emphasizes spatial effects and lends itself to physical interpretation more readily than the spectral-density functions. All of the information necessary to display the coherence function is available in the calculations performed in this paper. If the coherence function would have been analyzed, it may have led to a more detailed interpretation of the analytical results.

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[1]R. A. DANOFSKY, *Nucl. Sci. Eng.*, **36**, 28 (1969).

[2]J. R. SHEFF and R. W. ALBRECHT, *AEC Symposium Series No. 7* (1966).

[3]R. W. ALBRECHT and W. SEIFRITZ, *Nukleonik*, **11**, 143 (1968).

[4]W. SEIFRITZ and R. W. ALBRECHT, *Nukleonik*, **11**, 149 (1968).

[5]R. W. ALBRECHT and W. SEIFRITZ, *Japan-U.S. Seminar on Nuclear Reactor Noise Analysis*, pp. 285 (1968).

Further Comment on 'A Space-Dependent Reactor-Noise Formulation Utilizing Modal Expansions'

It has been pointed out by Albrecht[1] that there are fundamental differences between the auto-spectral-density function for a localized stochastic absorber and the cross-spectral-density function obtained for space-dependent observation points. In the case of a localized absorber, it would appear, from an analogy with the classical input-output noise relationship that the auto-spectral-density function is proportional to the magnitude of the space-dependent response of the reactor to a localized oscillating absorber. The characteristics of the cross-spectral-density function, however, are related to the space dependence of the observation points and noise sources.

"Sink frequencies" have been observed[2] for both of these cases and it has been suggested that it would be desirable to indicate the magnitudes of the auto-spectral-density function in the vicinity of the sink. These are reported in Table I. It has been noted[2] that the characteristics of the solutions for space points 4 and 5 show sensitivity at high frequencies to the number of modes used.

The real and imaginary parts of the cross-spectral-density function for space points 4 and 5 are shown in Table II. The real part changes signs at approximately

[1]R. W. ALBRECHT, *Nucl. Sci. Eng.*, **37**, 322 (1969).

[2]R. A. DANOFSKY, *Nucl. Sci. Eng.*, **36**, 28 (1969).

TABLE I

Magnitude of Auto-Spectral-Density Function for a Localized Absorber*

Frequency rad/sec	Space Point 4	Space Point 5
900	12.8	5.13
1000	8.13	2.94
2000	0.267	0.00124
3000	0.526	0.0231
4000	0.617	0.0256

*Tabulated value 10^3 times calculated value.

TABLE II

Real and Imaginary Parts of Cross-Spectral-Density Function*

Frequency rad/sec	Space Point 4		Space Point 5	
	Real	Imaginary	Real	Imaginary
200	1633	647.8	1081	225.8
300	460.7	348.2	333.7	135.5
400	123.9	202.7	115.1	89.60
500	11.16	125.2	39.07	63.20
600	-27.92	81.20	10.41	46.61
700	-40.09	54.71	-0.5946	35.46
800	-41.88	37.98	-4.567	27.56
900	-39.65	26.98	-5.647	21.73
1 000	-36.01	19.48	-5.549	17.31
2 000	-8.497	0.3338	-1.115	1.792
3 000	-0.7381	-0.8469	-0.3364	-0.2393
4 000	1.150	-0.6808	-0.1874	-0.4136
5 000	1.464	-0.4642	-0.1335	-0.3304
6 000	1.367	-0.3143	-0.1026	-0.2394
7 000	1.185	-0.2178	-0.08145	-0.1721
8 000	1.005	-0.1553	-0.06600	-0.1255
9 000	0.8514	-0.1139	-0.05438	-0.09341
10 000	0.7241	-0.08563	-0.04545	-0.07098

*Tabulated values 10^3 times calculated values.

600 rad/sec and again at 4000 rad/sec for space point 4 and at 700 rad/sec for space point 5. This frequency corresponds approximately to the sink frequency that has been observed for the coupled-core Argonaut reactor.[3,4] It would appear that the imaginary parts of the cross-spectral-density function may be particularly sensitive to the convergence of the modal solution since they are larger than might be expected for near symmetric locations.

The observation that the characteristics of the cross-spectral-density function are related to the degree of coupling of the fuel regions is certainly valid. An investigation of the effects of core spacing and the nuclear properties of the coupling region on the spectral functions is presently being performed.

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[3]R. A. HENDRICKSON and G. MURPHY, *Nucl. Sci. Eng.*, **31**, 215 (1968).

[4]R. W. ALBRECHT and W. SEIFRITZ, *Japan-U.S. Seminar on Nuclear Reactor Noise Analysis*, pp. 285,(1968).

On the Stabilizing Effect of Delayed Neutrons

For a nuclear reactor, the overall transfer function, which relates the Laplace transform of incremental power or neutron density $\delta n(s)$ to the Laplace transform of reactivity input $k(s)$, may be expressed as

$$\delta n(s)/[k(s)] = Z(s)/[1 + K(s)Z(s)] \ , \qquad (1)$$

provided that the block diagram shown in Fig. 1 represents the linear incremental model of the system. $Z(s)$, defined by

$$Z(s) = \frac{n_0}{l^* \left(s + \sum_i \frac{\beta_i s}{s + \lambda_i}\right)} \ ,$$

is the zero-power transfer function of the reactor, and $K(s)$ denotes the transfer function of the feedback block. Obviously, the operating power level or the neutron density of the reactor is indicated by n_0.

It is claimed by Smets[1] that delayed neutrons may exert a destabilizing effect upon a reactor system, if the system has an open-loop frequency characteristic which intersects the negative real axis twice in the form of curve A in Fig. 2a.

This conclusion was derived because curve A, which is the Nyquist plot of the system with the effects of delayed neutrons neglected, indicates a stable system, while curve B, obtained after the effects of delayed neutrons have been taken into account, reveals instability of the same power level.

It is true, at the power level n_0, the reactor without delayed neutrons is stable and the reactor with delayed neutrons is not. Such a result, however, is not sufficient to compare the degree of stability of the systems with the specified open-loop frequency characteristics, because, these are conditionally stable systems with two different stability regions.[2] Each system is stable for sufficiently low and sufficiently high power levels; that is, when both intersections of the Nyquist plot with the real axis are either to the right or to the left of the point $(-1 + j\ 0)$, it becomes unstable for a finite range of power between these two stability regions. Attention must be paid to the interesting fact that the range of power which corresponds to instability is different for each system. Therefore, the power level n_0 may lie in the instability region of the system with delayed neutrons, while it is within the stable power range of the system without delayed neutrons. At a different power level the results may reverse. For example, curves A′ and B′ in Fig. 2a indicate that at the power level $\frac{2}{3}\, n_0$, the system with delayed neutrons is stable, while the other system is not.

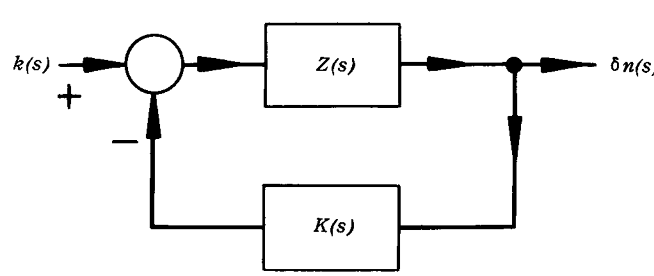

Fig. 1. Linear incremental model of a nuclear reactor.

[1]H. B. SMETS, *Nucl. Sci. Eng.*, **25**, 236 (1966).

[2]J. MILDA and N. SUDA, *Nucl. Sci. Eng.*, **11**, 55 (1960).