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Letters to the Editors

Divergence of the Mean Power Level During an Oscillation Experiment

The purpose of this letter is to present a simple formulation of the reactor dynamics equations for sinusoidal reactivity variation which can be solved analytically. It is thereby possible to estimate the rate of divergence of the power level during an oscillation experiment. Similarly, it is possible to predict a mean value of the reactivity less than zero which should eliminate the mean-power-level drift.

The usual reactor dynamics equations are:

$$\dot{n} = \frac{\rho - \beta}{\ell} n + \sum_i \lambda_i c_i \qquad (1)$$

$$\dot{c}_i = \frac{\beta_i n}{\ell} - \lambda_i c_i \,, \qquad (2)$$

where the notation is standard.

Let
$$\rho = 0 \qquad t < 0 \qquad (3)$$
$$\rho = \rho_1 + \rho_2 \sin \omega t \qquad t \geq 0$$
$$|\rho_1| < |\rho_2| << \beta .$$

The equations are scaled by defining a new set of variables,

$$N = n/n_0$$
$$C_i = c_i/c_{i0} \,, \qquad (4)$$

where n_0 and c_{i0} are the equilibrium values of n and c_i for $t < 0$.

Substitution of Eq. (4) into Eqs. (1), (2) and (3) yields

$$\tau_0 \dot{N} = (\epsilon_1 + \epsilon_2 \sin \omega t - 1) N + \sum_i \frac{\beta_i C_i}{\beta} \qquad (5)$$

$$\tau_i \dot{C}_i = N - C_i \,, \qquad (6)$$

where

$$\tau_0 = \ell/\beta$$
$$\tau_i = 1/\lambda_i$$
$$\epsilon_1 = \rho_1/\beta$$
$$\epsilon_2 = \rho_2/\beta$$

with initial conditions

$$N(t < 0) = C_i(t < 0) = 1.$$

In order to solve Eqs. (5) and (6), approximations are introduced that are accurate for low-frequency oscillation: the zero-prompt-lifetime approximation, $\tau_0 = 0$, and the replacement of the usual six delayed neutron groups by a single delayed group with constants, β^* and τ^*, assigned fictitious values such that the reactor transfer function is exactly represented at zero frequency by the approximations

$$\beta^* = \sum_i \beta_i$$
$$\tau^* = \frac{\sum_i \beta_i \tau_i}{\sum_i \beta_i} \,.$$

These approximations yield a good representation of the reactor transfer function, within the general limitation of neglecting spatial effects,

for $\qquad 0 \leq \frac{\omega}{2\pi} < \frac{1}{\tau_0}$ (footnote (a)).

In addition use of the zero-lifetime approximation requires that the reactivity never exceed prompt critical, a condition easily met in the oscillation experiment.

Introduction of these approximations reduces the original set of equations to the following:

$$N = C[1 - \epsilon_1 - \epsilon_2 \sin \omega t]^{-1} \qquad (7)$$

$$\tau^* \dot{C} = N - C \,. \qquad (8)$$

The solution for C from Eqs. (7) and (8) can be reduced to quadrature:

$$\omega \tau^* \ell n\, C = \epsilon_1 \int_0^{\omega t} \frac{d\xi}{1 - \epsilon_1 - \epsilon_2 \sin \xi} +$$
$$+ \epsilon_2 \int_0^{\omega t} \frac{d\xi \sin \xi}{1 - \epsilon_1 - \epsilon_2 \sin \xi} \,. \qquad (9)$$

The integrals in Eq. (9) can be evaluated to yield

[a] See, for example, M. A. Schultz, *Control of Nuclear Reactors and Power Plants*, (pages 115-120) McGraw-Hill, (1961).

$$\omega\tau^* \ell n\, C = -\omega t + \frac{2}{\sqrt{(1-\epsilon_1)^2-\epsilon_2^2}}\left[\tan^{-1}\left\{\frac{(1-\epsilon_1)\tan\frac{\omega t}{2}-\epsilon_2}{\sqrt{(1-\epsilon_1)^2-\epsilon_2^2}}\right\}+\right.$$

$$\left.-\tan^{-1}\left\{\frac{-\epsilon_2}{\sqrt{(1-\epsilon_1)^2-\epsilon_2^2}}\right\}\right] \qquad (10)$$

The value of $C(t)$ can be written more simply when $\omega t = (2n+1)\pi$. If we restrict consideration of the solution to only those values of t when the above equation is satisfied,

$$\omega\tau^* \ell n\, C = -\omega t + \frac{\omega t}{\sqrt{(1-\epsilon_1)^2-\epsilon_2^2}} \qquad (11)$$

Substitution of Eq. (11) into the equation for neutron density, Eq. (7), then yields

$$N(t) = \frac{1}{1-\epsilon_1-\epsilon_2 \sin\omega t}\exp\left\{\frac{t}{\tau^*}\left[\frac{1}{\sqrt{(1-\epsilon_1)^2-\epsilon_2^2}}-1\right]\right\} \qquad (12)$$

If ϵ_1 is chosen equal to zero (i.e., for reactivity oscillation about a zero mean value), the neutron flux can be approximated by

$$N(t) = [1+\epsilon_2 \sin\omega t]\exp\left\{\frac{\epsilon_2^2}{2}\frac{t}{\tau^*}\right\}. \qquad (13)$$

Hence, the mean power drift is exponential with an e- folding time of $2\tau^*/\epsilon_2^2$ and has a second-order dependence on the reactivity perturbation. It should be noted that the result is independent of the oscillation frequency, ω.

Evidently, the drift can be reduced to zero by setting

$$(1-\epsilon_1)^2-\epsilon_2^2 = 1$$

or

$$\epsilon_1 = 1-\sqrt{1+\epsilon_2^2} \approx -\frac{\epsilon_2^2}{2}$$

Although the simplicity of the model precludes rigorous application of the solution, the results are in agreement with physical intuition and show the important role of the delayed neutrons in causing the mean power drift during an oscillation-experiment.

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A More Straightforward Use of Variational Principles with Boundary Conditions

Pomraning and Clark[1] recently added boundary conditions to a variational principle for solutions to the transport equation[2] (the additional terms, although described as having to be guessed, were derived from a general procedure independently by Selengut[3]). Their treatment of one-group diffusion theory for a homogeneous slab will now be reviewed for the special situation in which scattering is isotropic and no neutrons enter the medium from outside its surfaces.

For a slab extending from $z=a$ to b, a functional is defined

$$F[\phi(z,\mu)] \equiv \int_a^b dz \int_{-1}^{1} d\mu\, [\phi^* H\phi - S\phi^* - T\phi] +$$

$$+\int_0^1 d\mu\, \mu\, \phi^*(a,\mu)\, \phi(a,\mu) - \int_{-1}^0 d\mu\, \mu\, \phi^*(b,\mu)\, \phi(b,\mu).$$

The operator H is given by

$$H\phi(z,\mu) \equiv \mu \frac{\partial\phi}{\partial z} + \phi - \frac{c}{2}\int_{-1}^{1} d\mu'\, \phi(z,\mu'),$$

where z is taken in units of total mean free paths. The symbol c is the number of secondaries per collision, S is the (given) source, and

$$\phi^*(z,\mu) \equiv \phi(z,-\mu),$$
$$T(z,\mu) \equiv S(z,-\mu).$$

Imposing the condition that F be stationary with respect to arbitrary variations in ϕ is equivalent to making ϕ equal the solution to the transport equation:

$$H\phi(z,\mu) = S(z,\mu) \text{ and}$$
$$\phi(a,\mu) = 0 \text{ for } \mu > 0.$$
$$\phi(b,\mu) = 0 \text{ for } \mu < 0.$$

We now further specialize to the situation in which S is isotropic:

$$S(z,\mu) = T(z,\mu) = \frac{1}{2} S_0(z).$$

The authors make a P-1 approximation,

$$\phi(z,\mu) = \frac{1}{2}\phi(z) + \frac{3}{2}\mu J(z),$$

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[1] G. C. POMRANING and M. CLARK, Jr., *Nucl. Sci. Eng.* **16**, 147-154 (1963).

[2] D. S. SELENGUT, *Hanford Quarterly Report*, HW-59126, 89-124, Richland, Wash., (1959).

[3] D. S. SELENGUT, *Trans. Am. Nucl. Soc.*, **5**, 1 (1962). p. 40.