used resolved parameters up to an energy of 4 keV, with an average gamma width of 0.0231 eV. The range between 4 keV and 32 keV was treated on a statistical basis. The contributions of each range are:

Energy (eV)	Resonance Integral (barns)
0.5 - 19.6	1.8
19.6 - 4060	77.7
4060 - 32000	0.7
Above 32000	1.1
p-wave capture	1.0
	82.3

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Statistical-Error Estimation for the Transfer-Function Measurements of a Noisy Reactor System

The application of the cross-correlation method in the determination of the dynamic response of a system which contains extraneous noise has long been accepted as one of the most reliable methods for recovering signals in the presence of noise. The theory of this method can be found in the literature[1,2,3]. Also, many discussions have been presented on the statistical errors of the results of measurements where some discrepancies appear because of the limitation in application of the theory[2,4]. In many situations, intuition plays a major role in estimating these statistical errors. Under the condition of using a sinusoidal input signal for transfer-function measurements, for a noisy reactor (e.g. EBWR) intuition has often led to the simple conclusion that the statistical error is dependent soley on the length of the record. In order to derive a more correct estimate of the error, the characteristics of the system noise and signal must be included in estimating the error.

The result of an investigation using statistical theory is presented here. The two most common types of noise have been used for illustration.

The assumptions that have been made are 1) the system noise is a stationary random process; 2) the ergodic hypothesis is valid; 3) the statistical error has a Gaussian distribution.

Let the system input be x, where

$$x = A \sin \omega t \,, \tag{1}$$

and let the system output be y, where

$$y = B \sin (\omega t + \phi) + n(t) \;. \tag{2}$$

The finite-time cross correlation between x and y is

$$\phi_{yx}(\tau) = \frac{1}{T}\int_0^T \Big[A \sin(\omega t + \omega\tau)\Big]\Big[B \sin(\omega t + \phi) + n(t)\Big]dt$$

$$= \frac{AB}{2} \cos(\omega\tau - \phi) + \epsilon(\tau, T) \,, \tag{3}$$

where $\epsilon(\tau, T)$ is the error function, and

$$\epsilon(\tau, T) = \frac{1}{T}\int_0^T A \sin(\omega t + \omega\tau) n(t)dt \;. \tag{4}$$

The standard deviation of ϵ can be found from the fourth moment[2] of functions x and n where

$$\epsilon^2(\tau, T) = \frac{2}{T^2}\int_0^T (T - \nu) \times$$

$$\times \Big[\Phi_{xx}(\nu)\, \Phi_{nn}(\nu) + \Phi_{xn}(\nu + \tau)\Phi_{nx}(\tau - \nu)\Big]d\nu$$

$$T = \frac{2k\pi}{\omega} \quad k = 0,1 \ldots \,, \tag{5}$$

where $\Phi_{xx}(\nu)$, $\Phi_{nn}(\nu)$, $\Phi_{xn}(\nu)$ and $\Phi_{nx}(\nu)$ are the theoretical correlation functions for x and n:

$$\Phi_{xn}(\nu) = \Phi_{nx}(\nu) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \times$$

$$\times\Big[A \sin \omega t\Big]\Big[n(t + \nu)\Big]dt = 0 \tag{6}$$

$$\Phi_{xx}(\nu) = \lim_{T\to\infty} \frac{1}{T}\int_0^T A^2 \sin \omega t \sin(\omega t + \omega\nu)dt$$

$$= \frac{A^2}{2} \cos\omega\nu \tag{7}$$

$$\Phi_{nn}(\nu) = \lim_{T\to\infty} \frac{1}{T}\int_0^T n(t)\, n(t + \nu)dt$$

$$= \sigma_n^2\; \Phi'_{nn}(\nu) \,, \tag{8}$$

[1]Y. W. LEE, *Statistical Theory of Communication,* John Wiley and Sons, Inc., New York, (1960).

[2]J. S. BENDAT, *Principles and Applications of Random Noise Theory,* John Wiley and Sons, Inc., New York, (1958).

[3]W. B. DAVENPORT and D. L. ROOT, *An Introduction to Theory of Random Signals and Noise,* McGraw-Hill Book Co., New York, (1958).

[4]V. RAJAGOPAL, "Experimental Study of Nuclear Reactor Internal Noise and Transfer Function Using Random Reactivity Variations and Correlation Analysis," (microfilm), University of Michigan, Ann Arbor, (1961).

where $\Phi'_{nn}(\nu)$ is the normalized autocorrelation function for $n(t)$, and σ_n^2 is the mean square value of $n(t)$.

Case I

$$\Phi'_{nn}(\nu) = e^{-a\nu} \tag{9}$$

Equation (5) becomes

$$\epsilon^2(\tau, T) = \frac{\sigma_n^2 A^2}{T^2(a^2+\omega^2)^2} \times \times \left[aT(a^2+\omega^2) + (\omega^2 - a^2)(1 - e^{-aT})\right]. \tag{10}$$

Since $aT \gg 1$ is usually encountered, equation (10) can be simplified as:

$$\epsilon^2(\tau, T) \simeq \frac{\sigma_n^2 A^2}{aT\left(1 + \frac{\omega^2}{a^2}\right)} . \tag{10-a}$$

These results show that the statistical error is a function of a, ω and T. A closer examination shows that the measurement would have larger errors at low frequencies and smaller errors at high frequencies.

Case II

$$\Phi'_{nn}(\nu) = e^{-a\nu} \cos \omega_0 \nu . \tag{11}$$

Equation (5) becomes

$$\epsilon^2(\tau, T) = \frac{\sigma_n^2 A^2}{2T^2} \left\{aT\left[\frac{1}{a^2+(\omega+\omega_0)^2} + \frac{1}{a^2+(\omega-\omega_0)^2}\right] - \frac{a^2-(\omega+\omega_0)^2}{\left[a^2+(\omega+\omega_0)^2\right]^2} - \frac{a^2-(\omega-\omega_0)^2}{\left[a^2+(\omega-\omega_0)^2\right]^2} + e^{-aT}\left[\frac{\cos(\omega_0 T+\Phi_1)}{a^2+(\omega+\omega_0)^2} + \frac{\cos(\omega_0 T+\Phi_2)}{a^2+(\omega-\omega_0)^2}\right]\right\}, \tag{12}$$

where

$$\Phi_1 = -\tan^{-1} \frac{2a(\omega+\omega_0)}{a^2+(\omega+\omega_0)^2} \tag{13}$$

$$\Phi_2 = +\tan^{-1} \frac{2a(\omega-\omega_0)}{a^2+(\omega-\omega_0)^2} . \tag{14}$$

Again, by using the condition that $aT \gg 1$, Eq. (12) can also be simplified as:

$$\epsilon^2(\tau, T) \simeq \frac{a\sigma_n^2 A^2}{2T^2}\left\{\frac{a^2+\omega^2+\omega_0^2}{\left[a^2+(\omega+\omega_0)^2\right]\left[a^2+(\omega-\omega_0)^2\right]}\right\}. \tag{12-a}$$

Equation (12-a) reduces to (10-a) when $\omega_0=0$. This shows that the largest error will occur when $\omega = \omega_0$ and for other frequencies the argument Case I applies.

In actual transfer function measurements, σ_n, a, and ω_0 can be measured beforehand by obtaining the autocorrelation function of the noise without sinusoidal signal. Then, for a given bound of $\epsilon^2(\tau, T)$, T. can be determined readily from the formula, or if knowing T, $\epsilon^2(\tau, T)$ can be calculated.

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