Letters to the Editor

Approximation to Neutron Escape Probability for Slab and Cylinder

In a recent Note,[1] an approximate expression of the neutron escape probability from an absorbing body was derived in the form

$$P(\tau) \approx \frac{1 - e^{-\tau}}{\tau} - A\tau e^{-\tau} = P_0(\tau) + P_A(\tau) \ , \qquad (1)$$

where τ is the optical mean-chord-length of the body and A is chosen so that the approximation be exact for $\tau = 1$. The validity of this approximation was demonstrated by the examples of the simplest geometries such as sphere, cylinder, and slab. The purpose of this Letter is improving the approximation for the special cases of slab and cylinder. Note that for a sphere, the evaluation of the exact expression of the escape probability does not require essentially more effort than that of the approximate one.) The idea behind the improvement is straightforward. We consider $A = A(\tau)$ as a function of τ instead of being constant, i.e., we write

$$P(\tau) \approx \frac{1 - e^{-\tau}}{\tau} - A(\tau)\tau e^{-\tau} \ . \qquad (2)$$

It is easy to show that if $A^{+}(\tau)$ is the function by the use of which in place of $A(\tau)$, Eq. (2) reproduces the exact escape probability $P(\tau)$, then

$$A^{+}(0) = -\frac{dP(0)}{d\tau} - \frac{1}{2} \ , \ \frac{dA^{+}(0)}{d\tau} < 0 \ ,$$

while

$$A^{+}(\tau) \propto \begin{cases} 4\ \exp(\tau/2)/\tau^3 & \text{for a slab ,} \\ 3\ \exp(\tau)/4\tau^4 & \text{for a cylinder ,} \end{cases}$$

as τ approaches infinity. Thus, a function, $A(\tau)$, decreasing for small τ values and increasing for large ones, may result in a better approximation to $P(\tau)$ than a constant. On the other hand, there is no need for a very accurate approximation to $A^{+}(\tau)$ for extremely small and large τ values, since in these cases the contribution of the correction term P_A to P is rather

TABLE I

Relative Errors of Different Approximations to the Escape Probability from an Infinite Slab of Optical Mean-Chord-Length, τ

τ	P_{exact}	Ref. 1 (ppt)[a]	FP[b] (ppt)	LS[c] (ppt)
0.2	0.8371	42.7	3.6	~0
0.4	0.7403	39.2	−2.0	−2.7
0.6	0.6665	27.1	−2.7	−0.1
0.8	0.6068	13.1	−1.6	1.5
1.0	0.5568	0.0	0.0	3.5
2.0	0.3903	−34.3	2.6	2.0
3.0	0.2955	−31.6	0.0	−4.1
4.0	0.2349	−19.2	−1.4	−6.0
5.0	0.1935	−9.0	−1.1	−4.8
6.0	0.1637	−3.1	−0.5	−2.8
7.0	0.1414	~0	0.4	−0.8
8.0	0.1243	0.9	~0	~0
9.0	0.1108	0.3	~0	~0
10.0	0.0998	0.8	0.4	~0

[a]Parts per thousand.
[b]FP ≡ three-point fitting.
[c]LS ≡ least-squares fitting.

TABLE II

Relative Errors of Different Approximations to the Escape Probability for an Infinite Cylinder of Optical Mean-Chord-Length, τ

τ	P_{exact}	Sauer (ppt)[a]	Ref. 1 (ppt)	FP[b] (ppt)	LS[c] (ppt)
0.2	0.88502	4.4	5.9	0.1	1.2
0.4	0.79303	4.1	6.1	0.0	−0.2
0.6	0.71649	2.5	4.3	0.4	−0.1
0.8	0.65162	0.6	2.1	0.8	0.2
1.0	0.59595	−1.2	0.0	1.2	0.5
2.0	0.40715	−5.6	−3.5	0.0	−0.6
3.0	0.30157	−4.7	1.6	−1.5	−1.7
4.0	0.23645	−2.3	7.5	0.0	~0
5.0	0.19323	−0.2	10.9	4.1	3.0
6.0	0.16286	1.3	11.9	5.6	5.6
7.0	0.14052	2.1	11.2	7.0	7.0
8.0	0.12346	2.6	10.0	7.4	7.4
9.0	0.11004	2.8	8.6	7.2	7.2
10.0	0.09923	2.7	7.3	6.5	6.5

[a]Parts per thousand.
[b]FP ≡ three-point fitting.
[c]LS ≡ least-squares fitting.

[1]Y. A. CHAO and A. S. MARTINEZ, *Nucl. Sci. Eng.*, **66**, 254 (1978).

small. Therefore, we do not try to find an approximate $A(\tau)$ function reproducing the divergence of the exact one at zero, in case of a slab, and at infinity, in both cases.

Numerical tests show that choosing $A(\tau)$ as a low-order polynomial of τ results in no substantial improvement. Thus, the simplest function having a slope similar to that of the exact $A^{+}(\tau)$ is

$$A(\tau) = A_1 \frac{1 + A_2\tau^2}{1 + A_3\tau} \quad , \tag{3}$$

where A_1, A_2, and A_3 are positive parameters to be fitted. Two methods of fitting were examined. In the first method, we demand that the approximation be exact for three given values of τ; in the second method, the mean-square-error of the approximation was minimized by means of the general purpose data evaluating code RFIT (Ref. 2), using the exact values of $P(\tau)$ at $\tau = 0.1\ (0.1)\ 1.0\ (1.0)\ 7.0$.

For slab geometry, the three fixed points (where the approximation is exact) are at $\tau = 0.3$, 1.0, and 3.0, and the resulting coefficients are

$$A_1 = 0.56253\ ,\ A_2 = 0.083450\ ,\ A_3 = 1.9768\ . \tag{4}$$

The least-squares (LS) fitting yields

$$A_1 = 0.63703\ ,\ A_2 = 0.11659\ ,\ A_3 = 2.5644\ . \tag{5}$$

In Table I, a comparison of the exact and approximate values of the escape probabilities is given through the relative errors due to the approximation of Eq. (1), with $A = 0.20474$ (Ref. 1), to the three-point fitting (FP), and to the LS of Eqs. (2) and (3) at several τ values.

For cylindrical geometry, the fixed points are $\tau = 0.4$, 2.0, and 4.0, and the coefficients are

$$A_1 = 0.14854\ ,\ A_2 = 0.14769\ ,\ A_3 = 0.76992\ , \tag{6}$$

while the LS fitting results in

$$A_1 = 0.14753\ ,\ A_2 = 0.13933\ ,\ A_3 = 0.72204\ . \tag{7}$$

Table II shows the relative errors of the approximations of Ref. 1 (with $A = 0.098323$) and of this Letter along with those of Sauer.[3]

Iván Lux
István Vidovszky

Central Research Institute for Physics
P.O. Box 49, H-1525 Budapest
Hungary

September 6, 1978

[2]Z. SZATMÁRY, "Data Evaluation Problems in Reactor Physics. Theory of Program RFIT," KFKI-1977-43, Central Research Institute for Physics, Budapest (1977).

[3]A. SAUER, *Nucl. Sci. Eng.*, **16**, 329 (1963).

Reply to the Comments by Lux and Vidovszky on an Approximation to Neutron Escape Probability

The original idea in Ref. 1 was to give a more careful analysis of the moment expansion approach and attempt to suggest a general approximation scheme for computing the neutron escape probability functions. To keep the parametrization simple (yet reasonably accurate), the effects from higher moments were only very crudely absorbed in the "effective" value of the second moment, A. To generalize A to a τ-dependent function[2] is to go beyond the second-moment approximation and include more detailed effects of higher moment terms. This flexibility of keeping a τ-dependent A has also been noticed by Carlvik[3] and by Chao and Yarbrough.[4]

The neutron escape probability is defined as

$$p(\tau) = \frac{1 - I(\tau)}{\tau}\ ,$$

$$I(\tau) = \int \exp(-\tau x) f(x)dx\ , \tag{1}$$

where $f(x)$ is the cord length distribution function and $x = l/\bar{l}$. From the general property of the Laplace transform, we see that the small τ behavior of $I(\tau)$ [and $p(\tau)$] is determined by the asymptotic behavior of $f(x)$ and that the asymptotic behavior of $I(\tau)$ [and $p(\tau)$] is determined by the small x behavior of $f(x)$. Although $f(x)$ is often very complicated, its limiting behavior in small and large x is often traceable. Because of the particular relation between $p(\tau)$ and $I(\tau)$, it is the small τ behavior of I that is more important in a practical approximation. In Ref. 1, we arrived at the approximation scheme of

$$I(\tau) = e^{-\tau}(1 + A\tau^2 + \ldots)\ . \tag{2}$$

Lux and Vidovszky remarked that no substantial improvement is attainable by generalizing A to a low-order polynomial of τ. This is misleading and is clarified below.

First, it should be pointed out that the relation given by Lux and Vidovszky,

$$\frac{dA^{+}(0)}{d\tau} < 0\ ,$$

does not seem to hold for a general case. In the other relation given by them,

$$A^{+}(0) = -\frac{dp(0)}{d\tau} - \frac{1}{2}\ ,$$

the derivative $dp(0)/d\tau$ may also be $-\infty$, and thus $A^{+}(0)$ is not always finite. As pointed out in Ref. 1, if $f(x)$ extends to infinity and does not drop off exponentially, then $A(\tau)$ has a singularity at $\tau = 0$. Depending on the nature of the singularity, $A^{+}(0)$ may diverge and $dA^{+}(0)/d\tau$ approaches $-\infty$. On the other hand, if $f(x)$ extends only to finite x or drops off exponentially, then $A^{+}(0)$ is finite and

$$\frac{dA^{+}(0)}{d\tau} = \left(-\frac{1}{3}\right) \cdot \langle (x-1)^3 \rangle\ .$$

Lux and Vidovszky's statement

$$\frac{dA^{+}(0)}{d\tau} < 0$$

implies that $\langle (x-1)^3 \rangle > 0$. I cannot see why this is true.

In the two specific cases of an infinite slab and infinite cylinder, $f(x)$ goes like $1/x^3$ and $1/x^4$, respectively, and $A(\tau)$ contains a $\ln\tau$ singularity in the former case and a $\tau\ln\tau$

[1]Y. A. CHAO and A. S. MARTINEZ, *Nucl. Sci. Eng.*, **66**, 254 (1978).

[2]I. LUX and I. VIDOVSZKY, *Nucl. Sci. Eng.*, **69**, 442 (1979).

[3]I. CARLVIK, Private Communication.

[4]Y. A. CHAO and M. YARBROUGH, "Application of the Moment Expansion Approximation to the Calculation of Dancoff Factors," in preparation.