Reply to "Comments on the Lyczkowski-Travis Drift-Flux Controversy"

The intent of the original Letter[1] on the Travis et al.[2] paper was twofold:

1. to develop the proper setting of the two-phase drift-flux approximation derived by these authors
2. to present an analysis showing a possible consequence of their approximation.

The criticism (if it can be called that) was meant to be constructive. We detect no controversy except in the title of Porsching's Letter.[3] The original analysis[1] shows that an initial relative velocity equal to a constant is an admissable solution of the equations and that this relative velocity would not change with time.

Porsching's analysis[3] is more general, since it shows that for the case of zero pressure gradients, an initially zero relative velocity will remain zero for all time even if the initial phase velocities are the same linear function of space. He also shows that if the phases are the same linear function of space but differ by a constant, that an initially nonzero relative velocity will change as time progresses.

We agree with Porsching's claim that Eq. (6) of Travis et al.[4] is incorrect. Consider the characteristic path

$$\frac{dx}{dt} = \bar{u} \ , \tag{1}$$

where $\bar{u} = \frac{1}{2}(u_p + u_f)$. Equation (1) implies the existence of level curves having values α such that

$$x = f(\alpha,t) \ . \tag{2}$$

One has to transform Eq. (4) of Travis et al.[4] from the coordinates (x,t) to the coordinates (α,t') where $t' = t$. A straightforward computation shows that

$$\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x} \equiv \frac{D}{Dt} = \frac{\partial}{\partial t'} \ , \tag{3}$$

and that

$$\frac{\partial}{\partial x} = \frac{1}{f_\alpha}\frac{\partial}{\partial \alpha} \ , \tag{4}$$

where

$$f_\alpha = \left(\frac{\partial f}{\partial \alpha}\right)_{t'=t} \ , \tag{4a}$$

Therefore, Eq. (4) of Ref. 4 transforms into

$$\frac{1}{u_r}\frac{Du_r}{Dt} = -\frac{1}{f_\alpha}\frac{\partial \bar{u}}{\partial \alpha} \ , \tag{5}$$

where $u_r = u_p - u_f$. It is also easy to show that the right side of Eq. (5) above is also equal to

$$-\frac{1}{f_\alpha}\frac{\partial \bar{u}}{\partial \alpha} = -\frac{1}{\bar{u}}\left(\frac{D\bar{u}}{Dt} - \frac{\partial \bar{u}}{\partial t}\right) \ . \tag{6}$$

Hence, Eq. (6) of Travis et al.[4] results only when $\partial\bar{u}/\partial t = 0$, and we agree with Eq. (12) of Porsching.[3] Since Eq. (6) of the Travis et al. reply is incorrect, the conclusions following that equation are also incorrect.

Equation (5) above may be integrated from time t_0 to time t_1 as

$$\ln\left(\frac{|u_r|_{t_1}}{|u_r|_{t_0}}\right) = -\int_{t_0}^{t_1} \frac{1}{f_\alpha}\frac{\partial \bar{u}}{\partial \alpha}(t')dt' \ . \tag{7}$$

Clearly, this is as far as we can go in the analysis without closing the system of equations. It is more advantageous, then, to work with the original equations under consideration in the form

$$\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial x} = K \ , \tag{8}$$

and

$$\frac{\partial u_f}{\partial t} + u_f\frac{\partial u_f}{\partial x} = K \ , \tag{8a}$$

where K is assumed to be a constant in order to perform an analysis more general than Porsching's.[4]

Let s_p be the arc length along

$$\frac{dx}{dt} = u_p \ , \tag{9}$$

and s_f be the arc length along

$$\frac{dx}{dt} = u_f \ . \tag{9a}$$

Equations (8) through (9a) can be rewritten as

$$\frac{du_i}{ds_i} = K \ , \tag{10}$$

$$\frac{dx_i}{ds_i} = \frac{u_i}{[1 + (u_i)^2]^{1/2}} \ , \tag{11}$$

and

$$\frac{dx_i}{ds_i} = \frac{1}{[1 + (u_i)^2]^{1/2}} \ , \quad i = p,f \ . \tag{12}$$

Let the initial velocities be prescribed as

$$u_i = u_{io}(x,t) \ , \tag{13}$$

along the initial data curve

$$s_o = s_o(x,t) \ . \tag{14}$$

Equation (10) can be integrated as

$$u_i = Ks_{io} + u_{io} \ , \tag{15}$$

where $s_{io} = s_i - s_o$, the arc length from the initial data curve. Equation (15) allows Eqs. (11) and (12) to be integrated to determine the trajectories of u_i as

$$x_i = \int_{s_o}^{s_i} \frac{Ks_{io} + u_{io}}{[1 + (Ks_{io} + u_{io})^2]^{1/2}} ds_i + x_{io} \ , \tag{16}$$

and

$$t_i = \int_{s_o}^{s_i} \frac{ds_i}{[1 + (Ks_{io} + u_{io})^2]^{1/2}} + t_{io} \ . \tag{17}$$

The situation is now clearer. If the phases start at the same point in space and have the same functional initial values, then the trajectories are the same, the arc lengths from the initial data curve are the same and the initial relative velocity never changes as time progresses. Otherwise, the relative velocity does change from the initial value. Most transients

[1] R. W. LYCZKOWSKI, *Nucl. Sci. Eng.*, **71**, 77 (1979).

[2] J. R. TRAVIS, F. H. HARLOW, and A. A. AMSDEN, *Nucl. Sci. Eng.*, **61**, 1 (1976).

[3] T. A. PORSCHING, *Nucl. Sci. Eng.*, **73**, 304 (1980).

[4] J. R. TRAVIS, W. C. RIVARD, and F. H. HARLOW, *Nucl. Sci. Eng.*, **71**, 79 (1979).

are started with the two phases at rest or are formed at equal velocities.

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