distinguish among bodies of different geometries, since it does not depend on the detailed shape of the chord distribution function,[4] $f(l)$, but only on the mean chord length $\bar{l} = 4V/S$. But this approximation gives an error of 18% for intermediate values of $\Sigma\bar{l}$ in the case of solid cylinders. It was guessed that if a polynomial is expressed in terms of $\Sigma\bar{l}(1 + \Sigma\bar{l})$ we may be able to get an expression where only the coefficients will depend on the shape of the geometry and the above-mentioned polynomials were obtained for the simple geometries of sphere, slab, and infinite solid cylinder.

The polynomials for various geometries can be put in the form

$$P = G_0 + G_1X + \ldots + G_9X^9 \ ,$$

where the G's are the coefficients. For all three geometries the values are given in Table I, and X is expressed as

$$X = \frac{\Sigma\bar{l}}{1 + \Sigma\bar{l}} \ .$$

The results for all three geometries are given in Tables II, III, and IV. It is observed that, using these polynomials, we get results most of which agree up to the fourth decimal place of the exact results.

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[4]A. SAUER, *Nucl. Sci. Eng.*, **16**, 329 (1963).

Reply to "Polynomial Expression for the Neutron Escape Probability from an Absorbing Body"

I have three comments on the Letter by Raghav[1]:

1. The polynomial suggested in the Letter,

$$p = \sum_{n=0} G_n X^n \ , \tag{1}$$

where

$$X = \Sigma\bar{l}/(1 + \Sigma\bar{l}) \ , \tag{2}$$

does not satisfy the (exact) limiting behavior of

$$p = \begin{cases} 1 & \text{as } \Sigma\bar{l} \to 0 \\ 1/\Sigma\bar{l} & \text{as } \Sigma\bar{l} \to \infty \end{cases} \ , \tag{3}$$

which is crucial for the Wigner et al.[2] rational approximation. For example, at $\Sigma\bar{l} = 0$, the error in Eq. (1) is $(G_0 - 1)$, or ~2 to 4%, according to Table I of the Letter. In fact, this polynomial approximation, in a more satisfactory representation than Eq. (1), can be derived in the following way. In the exact expression for the escape probability

$$p = \left[1 - \int \exp(-\Sigma l) f(l) dl\right] \Big/ \Sigma\bar{l} \ , \tag{4}$$

if the exponential factor in the integral, $\exp(-\Sigma l)$, is approximated by the rational function $1/(1 + \Sigma l)$, we have

$$p \cong \left[1 - \int \frac{f(l)}{1 + \Sigma l} f(l) dl\right] \Big/ \Sigma\bar{l} \ , \tag{5}$$

which still satisfies the conditions of Eq. (3). Now we can make the same moment expansion approximation suggested in Ref. 3 by expanding $1/(1 + \Sigma l)$ in the integral in a power series around $l = \bar{l}$. This leads Eq. (5) to

$$p \cong \frac{1}{1 + \Sigma\bar{l}} + \left(\frac{1}{1 + \Sigma\bar{l}}\right)^2 \left[\sum_{n=1}^{\infty} A_n \left(\frac{\Sigma\bar{l}}{1 + \Sigma\bar{l}}\right)^n\right] \ , \tag{6a}$$

or

$$p \cong (1 - x) + (1 - x)^2 \left(\sum_{n=1} A_n x^n\right) \ , \tag{6b}$$

which again satisfies the limiting behavior of Eq. (3). The polynomial of Eq. (6) is, of course, the same as that of Eq. (1), provided some restrictions interrelating G_n are imposed on Eq. (1). I believe that if the expression Eq. (6) is adopted, the least-squares fit in the Letter will be substantially improved because it gets rid of the unnecessary correlations among the coefficients, and the coefficients A_n will also assume more systematic values than G_n do. Although this derivation relies on the rational approximation to the integrand, the representation, Eq. (6), itself can be regarded as being independent of the assumption since the coefficients are practically determined by fitting anyway.

2. I have recently considered this polynomial approximation in my work of extending the fast reactor Bondarenko formalism to thermal reactors. One crucial question involved there is the preservation of the equivalence relation when the Wigner et al. rational approximation is improved. It turns out that Eq. (6) is very useful for resolving that difficulty.

3. For the same reason given in my Reply[4] to the letter by Lux and Vidovszky,[5] inclusion of terms involving $X\ln X$ may improve the accuracy of Eq. (6) with less numbers of adjustable coefficients. But such a term is not good for the Bondarenko work discussed in my second comment.

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[1]HEM PRABHA RAGHAV, *Nucl. Sci. Eng.*, **73**, 302 (1980).

[2]E. P. WIGNER, E. CREUTZ, H. JUPNIK, and T. SNYDER, *J. Appl. Phys.*, **26**, 260 (1955).

[3]Y. A. CHAO and A. S. MARTINEZ, *Nucl. Sci. Eng.*, **66**, 254 (1978).

[4]Y. A. CHAO, *Nucl. Sci. Eng.*, **69**, 443 (1979).

[5]I. LUX and I. VIDOVSZKY, *Nucl. Sci. Eng.*, **68**, 442 (1979).

Comments on the Lyczkowski-Travis Drift-Flux Controversy

The literature on two-phase flow models is replete with questions concerning the validity of the defining mathematical systems.[1-3] Naturally, this has been a source of confusion for engineers and scientists attempting to study two-phase flow phenomena analytically and numerically. The recent communication by Lyczkowski[4] criticizing a drift-flux approximation proposal by Travis et al.,[5] and the subsequent reply[6] unnecessarily increase this confusion. Lyczkowski's criticism is inaccurate, but *not* for the reason given in the Travis reply.

According to Lyczkowski, the relative velocity $(u_p - u_f)$ predicted by Ref. 5 is constant. The supporting argument proceeds from the equation

$$\frac{\partial}{\partial t}(u_p - u_f) + \frac{\partial}{\partial x}\left[\frac{1}{2}(u_p^2 - u_f^2)\right] = 0 \ , \qquad (1)$$

which is obtained by subtraction of

$$\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial x} = g_x - \frac{1}{\rho_t}\frac{\partial p}{\partial x} \ , \qquad (2)$$

from

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} = g_x - \frac{1}{\rho_t}\frac{\partial p}{\partial x} \ . \qquad (3)$$

Now Eqs. (2) and (3) are the same equation! Hence, assuming that the initial value problem

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = g_x - \frac{1}{\rho_t}\frac{\partial p}{\partial x} \ , \quad t > 0 \ , \qquad (4)$$

$$u(x,0) = u_0(x) \ , \qquad (5)$$

is properly posed [we presume that the larger problem, which includes a defining relationship for p, is properly posed, so that in Eq. (4) p can be regarded as data], it follows that u_p will differ from u_f only by virtue of the initial condition, Eq. (5). This implies that if the relative velocity is zero at $t = 0$, then it will subsequently remain zero. However, a constant nonzero relative velocity at $t = 0$ will not necessarily remain constant for $t > 0$.

These remarks are illustrated by the following particular case of Eqs. (4) and (5):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \ , \quad t > 0 \ , \qquad (4')$$

$$u(x,0) = x + c \ , \qquad (5')$$

where c is an arbitrary constant. One easily verifies that the solution of Eqs. (4′) and (5′) is

$$u(x,t) = (x + c)/(t + 1) \ . \qquad (6)$$

Thus, if c_f and c_p are constants, and if

$$u_f(x,t) \equiv (x + c_f)/(t + 1) \ , \qquad (7)$$

$$u_p(x,t) \equiv (x + c_p)/(t + 1) \ , \qquad (8)$$

then the relative velocity,

$$u_p(x,t) - u_f(x,t) = (c_p - c_f)/(t + 1) \ ,$$

satisfies Eq. (1), and the *constant* initial condition

$$u_p(x,0) - u_f(x,0) = c_p - c_f \ .$$

However, $(u_p - u_f)$ clearly does not remain constant unless $c_p = c_f$.

In their reply, Travis et al.[6] purport to refute Lyczkowski by solving Eq. (1) by the "method of Lagrange." Thus, defining $u_r = u_p - u_f$ and $\bar{u} = \frac{1}{2}(u_p + u_f)$, they rewrite Eq. (1) as

$$\frac{\partial u_r}{\partial t} + \bar{u}\frac{\partial u_r}{\partial x} = -u_r\frac{\partial \bar{u}}{\partial x} \ . \qquad (9)$$

Then along the characteristic $x(t)$ given by

$$\frac{dx}{dt} = \bar{u} \ , \qquad (10)$$

they claim that Eq. (9) reduces to

$$\frac{d\bar{u}}{\bar{u}} + \frac{du_r}{u_r} = 0 \ . \qquad (11)$$

But Eq. (11) is incorrect! Along the characteristic of Eq. (10), we have

$$\frac{d\bar{u}}{dt} = \frac{\partial \bar{u}}{\partial x}\bar{u} + \frac{\partial \bar{u}}{\partial t} \ .$$

Therefore, Eq. (9) becomes

$$\frac{1}{u_r}\frac{du_r}{dt} + \frac{1}{\bar{u}}\left[\frac{d\bar{u}}{dt} - \frac{\partial \bar{u}}{\partial t}\right] = 0 \ , \qquad (12)$$

which reduces to Eq. (11) only if $\partial\bar{u}/\partial t = 0$.

Returning to Eqs. (7) and (8), we see that

$$\bar{u}(x,t) = \frac{x + \frac{1}{2}(c_f + c_p)}{t + 1} \equiv \frac{x + \bar{c}}{t + 1} \ .$$

Thus, Eq. (10) has the general solution

$$x(t) = c(t + 1) - \bar{c} \ ,$$

and along this curve

$$\frac{d\bar{u}}{dt} = \frac{d^2x}{dt^2} = 0 \ .$$

Since also along this characteristic,

$$\frac{1}{u_r}\frac{du_r}{dt} = -\frac{1}{t + 1} \ ,$$

$$\frac{\partial \bar{u}}{\partial t} = -\frac{x + \bar{c}}{(t + 1)^2} \ ,$$

it follows that Eq. (12) is satisfied, but Eq. (11) is not.

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