factors for the published buildup factors are welcome in that they provide information consistent with the American National (ANSI) Standard. The question in regard to the "dose equivalent index," H_I, is more profound. This quantity, the maximum dose equivalent within a 30-cm-diam tissue sphere, was defined by the International Commission on Radiation Units and Measurements in 1971, but it appears to us that the industry has not as yet reached a consensus as to its application. Because the expression of the industry's position is an important input in the course of the development and approval of a standard, this question will be considered when this ANSI standard (ANSI/ANS-6.1.1-1977) is reviewed. By then, conversion factors based on the recommendations of the National Committee on Radiation Protection and Measurement or other competent authority should be available to those involved in the review and revision of the standard.

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Polynomial Expression for the Neutron Escape Probability from an Absorbing Body

In a recent paper,[1] an approximate expression for the escape probability was derived in the form

$$P(\tau) = \frac{1 - e^{-\tau}}{\tau} - A\tau e^{-\tau} = P_0(\tau) + P_A(\tau) \ ,$$

where τ is the optical mean chord length of the body and A is chosen so that the approximation be exact for $\tau = 1$. The validity of this approximation was demonstrated by the examples of the simplest geometries such as sphere, slab, and cylinder. Then, Lux et al.[2] considered $A = A(\tau)$ as a function of τ instead of being constant. In this way, they further improved the results when compared with exact results for slab and cylinder. After all these improvements, the maximum error in the probabilities is ~0.7%.

Since the neutron escape probability from an absorbing body plays a very important role in the reactor physics calculations, the need arises to calculate this probability as accurately as possible without spending much computer time. An effort was made in this direction, and it was found that this probability can be expressed in terms of a polynomial. This expression is in terms of $\Sigma\bar{l}/(1 + \Sigma\bar{l})$, where Σ is the total macroscopic cross section and $\bar{l}$ is the mean chord length of

[1] Y. A. CHAO and A. S. MARTINEZ, *Nucl. Sci. Eng.*, **66**, 254 (1978).

[2] I. LUX and I. VIDOVSZKY, *Nucl. Sci. Eng.*, **69**, 442 (1979).

TABLE I

The Coefficients for Sphere, Slab, and Infinite Solid Cylinder

Coefficients	Sphere	Slab	Cylinder
G_0	1.018916	0.981945	0.964132
G_1	−0.783758	−0.916012	−0.226278
G_2	0.536125	0.354657	−2.153280
G_3	−1.008946	−0.138498	4.240849
G_4	−3.336077	−1.177880	−5.079923
G_5	9.212776	1.340848	2.261080
G_6	−5.177525	−0.513780	0
G_7	−8.921067	0.842021	0
G_8	13.664571	−1.977204	0
G_9	−5.207047	1.211139	0

TABLE II

Relative Errors (%) of Different Approximations to the Escape Probability in the Case of a Sphere

τ	P(exact)	Wigner et al.	Reference 1[a]	Reference 1[b]	Polynomial
0.2	0.8960	−7.0	0.01	−0.01	0.10
0.4	0.8069	−11.40	0.07	−0.13	~0
0.6	0.7304	−14.43	0.14	−0.13	0
0.8	0.6643	−16.36	0.24	−0.08	0
1.0	0.6070	−17.63	0.35	0.00	0
2.0	0.4110	−18.79	1.07	0.69	0
3.0	0.3024	−17.33	1.65	1.36	0
4.0	0.2363	−15.36	1.92	1.74	0
5.0	0.1929	−13.58	1.89	1.79	0
6.0	0.1626	−12.12	1.68	1.62	0
7.0	0.1403	−10.91	1.45	1.42	0
8.0	0.1233	−9.89	1.21	1.21	0
9.0	0.1099	−9.01	1.03	1.03	0
10.0	0.0991	−8.27	0.87	0.87	0

[a] $A = 0.0625$.
[b] $A = 0.0684$.

the body. The coefficients depend on the shape of the geometry. Polynomials have been obtained for sphere, slab, and infinite solid cylinder. The coefficients are obtained by the least-squares fit method. This fitting is done at some points, and the results are compared at other points.

The reason for expressing this polynomial in terms of $\Sigma\bar{l}/(1 + \Sigma\bar{l})$ is that this factor is nothing but the Wigner et al.[3] expression for collision probability in an absorbing body. In their classical paper on the theory of resonance absorption, Wigner et al. introduced for the escape probability from a body with uniform source density, the expression

$$P = \frac{1}{\Sigma\bar{l}}\left[1 - \int dl\, f(l)\exp(-\Sigma l)\right] \approx \frac{1}{1 + \Sigma\bar{l}} ,$$

which tends to correct the limit for small and for large values of $\Sigma\bar{l}$. Here Σ is the total cross section and $f(l)$ is the chord distribution function. This rational approximation does not

[3] E. P. WIGNER, E. CREUTZ, H. JUPNIK, and T. SNYDER, *J. Appl. Phys.*, **26**, 260 (1955).

TABLE III

Relative Errors (%) of Different Approximations to the Escape Probability for the Case of an Infinite Slab

τ	P(exact)	Wigner et al.	Reference 1	Reference 2		Polynomial
				FP[a]	LS[b]	
0.2	0.8371	−0.45	4.27	0.36	0	0.07
0.4	0.7403	−3.51	3.92	−0.20	−0.27	0.1
0.6	0.6665	−6.23	2.71	−0.27	−0.01	0.04
0.8	0.6068	−8.44	1.31	−0.16	0.15	~0
1.0	0.5568	−10.20	0.00	0.00	0.35	~0
2.0	0.3903	−14.60	−3.43	−0.26	0.20	~0
3.0	0.2955	−15.40	−3.16	0.00	−0.41	~0
4.0	0.2349	−14.86	−1.91	−0.14	−0.60	~0
5.0	0.1935	−13.85	−0.90	−0.11	−0.48	~0
6.0	0.1637	−12.71	−0.30	−0.05	−0.28	~0
7.0	0.1414	−11.60	~0	0.04	−0.08	~0
8.0	0.1243	−10.62	~0	~0	~0	~0
9.0	0.1108	−9.75	~0	~0	~0	~0
10.0	0.09908	−8.92	~0	0.04	~0	~0

[a] Approximation is exact for three fixed points.
[b] Least-squares fitting to get $A(\tau)$.

TABLE IV

Relative Errors (%) of Different Approximations to the Escape Probability for the Case of an Infinite Cylinder

τ	P(exact)	Wigner et al.	Sauer	Reference 1	Reference 2		Polynomial
					FP[a]	LS[b]	
0.2	0.8850	−5.84	0.44	0.59	0.01	0.12	0.027
0.4	0.7930	−9.92	0.41	0.61	0.00	−0.02	~0
0.6	0.7165	−12.77	0.25	0.43	0.04	−0.01	0
0.8	0.6516	−14.72	0.06	0.21	0.08	0.02	0
1.0	0.5960	−16.11	−0.12	0.00	0.12	0.05	0
2.0	0.4072	−18.15	−0.56	−0.36	0.00	−0.06	0
3.0	0.3016	−17.11	−0.47	0.15	−0.15	−0.17	0
4.0	0.2365	−15.43	−0.23	0.73	0.00	~0	0
5.0	0.1932	−13.72	−0.02	1.11	0.41	0.30	0
6.0	0.1629	−12.28	0.13	1.16	0.56	0.56	0
7.0	0.1405	−11.03	0.21	1.14	0.70	0.70	0
8.0	0.1235	−10.04	0.26	0.97	0.74	0.74	0
9.0	0.1100	−9.09	0.28	0.95	0.72	0.72	0
10.0	0.0992	−8.37	0.27	0.81	0.65	0.65	0

[a] Approximation is exact for three fixed points.
[b] Least-squares fitting to get $A(\tau)$.

distinguish among bodies of different geometries, since it does not depend on the detailed shape of the chord distribution function,[4] $f(l)$, but only on the mean chord length $\bar{l} = 4V/S$. But this approximation gives an error of 18% for intermediate values of $\Sigma\bar{l}$ in the case of solid cylinders. It was guessed that if a polynomial is expressed in terms of $\Sigma\bar{l}(1 + \Sigma\bar{l})$ we may be able to get an expression where only the coefficients will depend on the shape of the geometry and the above-mentioned polynomials were obtained for the simple geometries of sphere, slab, and infinite solid cylinder.

The polynomials for various geometries can be put in the form

$$P = G_0 + G_1X + \ldots + G_9X^9 \ ,$$

where the G's are the coefficients. For all three geometries the values are given in Table I, and X is expressed as

$$X = \frac{\Sigma\bar{l}}{1 + \Sigma\bar{l}} \ .$$

The results for all three geometries are given in Tables II, III, and IV. It is observed that, using these polynomials, we get results most of which agree up to the fourth decimal place of the exact results.

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[4]A. SAUER, *Nucl. Sci. Eng.*, **16**, 329 (1963).

Reply to "Polynomial Expression for the Neutron Escape Probability from an Absorbing Body"

I have three comments on the Letter by Raghav[1]:

1. The polynomial suggested in the Letter,

$$p = \sum_{n=0} G_n X^n \ , \tag{1}$$

where

$$X = \Sigma\bar{l}/(1 + \Sigma\bar{l}) \ , \tag{2}$$

does not satisfy the (exact) limiting behavior of

$$p = \begin{cases} 1 & \text{as} \quad \Sigma\bar{l} \to 0 \\ 1/\Sigma\bar{l} & \text{as} \quad \Sigma\bar{l} \to \infty \end{cases} \ , \tag{3}$$

which is crucial for the Wigner et al.[2] rational approximation. For example, at $\Sigma\bar{l} = 0$, the error in Eq. (1) is $(G_0 - 1)$, or ~2 to 4%, according to Table I of the Letter. In fact, this polynomial approximation, in a more satisfactory representation than Eq. (1), can be derived in the following way. In the exact expression for the escape probability

$$p = \left[1 - \int \exp(-\Sigma l) f(l)\, dl\right] \Big/ \Sigma\bar{l} \ , \tag{4}$$

if the exponential factor in the integral, $\exp(-\Sigma l)$, is approximated by the rational function $1/(1 + \Sigma l)$, we have

$$p \cong \left[1 - \int \frac{f(l)}{1 + \Sigma l} f(l)\, dl\right] \Big/ \Sigma\bar{l} \ , \tag{5}$$

which still satisfies the conditions of Eq. (3). Now we can make the same moment expansion approximation suggested in Ref. 3 by expanding $1/(1 + \Sigma l)$ in the integral in a power series around $l = \bar{l}$. This leads Eq. (5) to

$$p \cong \frac{1}{1 + \Sigma\bar{l}} + \left(\frac{1}{1 + \Sigma\bar{l}}\right)^2 \left[\sum_{n=1}^{\infty} A_n \left(\frac{\Sigma\bar{l}}{1 + \Sigma\bar{l}}\right)^n\right] \ , \tag{6a}$$

or

$$p \cong (1 - x) + (1 - x)^2 \left(\sum_{n=1} A_n x^n\right) \ , \tag{6b}$$

which again satisfies the limiting behavior of Eq. (3). The polynomial of Eq. (6) is, of course, the same as that of Eq. (1), provided some restrictions interrelating G_n are imposed on Eq. (1). I believe that if the expression Eq. (6) is adopted, the least-squares fit in the Letter will be substantially improved because it gets rid of the unnecessary correlations among the coefficients, and the coefficients A_n will also assume more systematic values than G_n do. Although this derivation relies on the rational approximation to the integrand, the representation, Eq. (6), itself can be regarded as being independent of the assumption since the coefficients are practically determined by fitting anyway.

2. I have recently considered this polynomial approximation in my work of extending the fast reactor Bondarenko formalism to thermal reactors. One crucial question involved there is the preservation of the equivalence relation when the Wigner et al. rational approximation is improved. It turns out that Eq. (6) is very useful for resolving that difficulty.

3. For the same reason given in my Reply[4] to the letter by Lux and Vidovszky,[5] inclusion of terms involving $X\ln X$ may improve the accuracy of Eq. (6) with less numbers of adjustable coefficients. But such a term is not good for the Bondarenko work discussed in my second comment.

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[1]HEM PRABHA RAGHAV, *Nucl. Sci. Eng.*, **73**, 302 (1980).

[2]E. P. WIGNER, E. CREUTZ, H. JUPNIK, and T. SNYDER, *J. Appl. Phys.*, **26**, 260 (1955).

[3]Y. A. CHAO and A. S. MARTINEZ, *Nucl. Sci. Eng.*, **66**, 254 (1978).

[4]Y. A. CHAO, *Nucl. Sci. Eng.*, **69**, 443 (1979).

[5]I. LUX and I. VIDOVSZKY, *Nucl. Sci. Eng.*, **68**, 442 (1979).

Comments on the Lyczkowski-Travis Drift-Flux Controversy

The literature on two-phase flow models is replete with questions concerning the validity of the defining mathematical