

In Eq. (4), we must add the prescription that doubly Cauchy integrals (which will appear when operating with Eq. (4)) are to be evaluated by interchange of integration order without regard to the dictates of the Bertrand-Poincaré' transformation. Thus, in using Eq. (4), one must employ the definition

$$\int_{-1}^{+1} \frac{d\mu}{\nu - \mu} \int_{-1}^{+1} \frac{F(\mu, \nu')}{\nu' - \mu} d\nu' = \int_{-1}^{+1} d\nu' \int_{-1}^{+1} \frac{F(\mu, \nu')}{(\nu - \mu)(\nu' - \mu)} d\mu, \quad (5)$$

where $F(\mu, \nu)$ satisfies a Hölder condition in the interval $(-1, +1)$. Of course, Eq. (5) is in conflict with the Bertrand-Poincaré' formula⁵

$$\begin{aligned} & \int_{-1}^{+1} \frac{d\mu}{\nu - \mu} \int_{-1}^{+1} \frac{F(\mu, \nu')}{\nu' - \mu} d\nu' \\ &= \int_{-1}^{+1} d\nu' \int_{-1}^{+1} \frac{F(\mu, \nu')}{(\nu - \mu)(\nu' - \mu)} d\mu + \pi^2 F(\nu, \nu). \quad (6) \end{aligned}$$

Using the closure condition of Eq. (4) in Eq. (3) we note that $\Delta(\mu, \mu_0) = 0$ and thus previous results are in agreement with Eq. (3). We also note that the closure condition is intimately linked with the term representing the uncollided flux. This is to be expected since the unusual functional properties of the angular Green's function are found in the uncollided term.

It is not difficult to find a closure condition for the function set $\{\phi(\pm L, \mu), \phi(\nu, \mu)\}$ that satisfies the 'ordinary' rules of integration as expressed in Eq. (6). The result is

$$\begin{aligned} \frac{\mu^2 \lambda^2(\mu)}{M(\mu)} \delta(\mu - \mu') &= \frac{\mu \mu' \phi(L, \mu) \phi(L, \mu')}{M_+} + \\ &+ \frac{\mu \mu' \phi(-L, \mu) \phi(-L, \mu')}{M_-} + \\ &+ \mu \mu' \int_{-1}^{+1} \frac{\phi(\nu, \mu) \phi(\nu, \mu')}{M(\nu)} d\nu, \quad (7) \end{aligned}$$

where $\lambda(\mu)$ is given in Ref. 2. Using Eq. (7) in Eq. (3) yields the angular Green's function

$$\begin{aligned} \Psi_G(x, \mu; \mu_0) &= \frac{\phi(L, \mu) \phi(L, \mu_0)}{M_+} e^{-x/L} + \\ &+ \int_0^{+1} \frac{\phi(\nu, \mu) \phi(\nu, \mu_0)}{M(\nu)} e^{-x/\nu} d\nu + \\ &+ h(\mu) \delta(\mu - \mu_0) \left(\frac{\pi c \mu}{2}\right)^2 \frac{e^{-x/\mu}}{M(\mu)}, \quad x > 0 \\ &= - \frac{\phi(-L, \mu) \phi(-L, \mu_0)}{M_-} e^{x/L} - \\ &- \int_{-1}^0 \frac{\phi(\nu, \mu) \phi(\nu, \mu_0)}{M(\nu)} e^{-x/\nu} d\nu - \\ &- h(-\mu) \delta(\mu - \mu_0) \left(\frac{\pi c \mu}{2}\right)^2 \frac{e^{-x/\mu}}{M(\mu)}, \quad x < 0. \quad (8) \end{aligned}$$

With this functional we need not include any added prescriptions such as the rule of Eq. (5).

Let us also point out that one can approach the problem of determining the angular Green's function by considering a distributed source of the form $S(\mu)\delta(x)$, where $S(\mu)$ satisfies a Hölder condition in the interval $(-1, +1)$. The solution is put in the form

$$\Psi(x, \mu) = \int_{-1}^{+1} S(\mu_0) \Psi_G(x, \mu; \mu_0) d\mu_0. \quad (9)$$

If the rules of integrating Cauchy singular functions are followed (esp. Eq. (6)), then the Green's function which results is that given by Eq. (8).

In conclusion, we note that the angular Green's functions which appear in the literature require a further prescription (as given in Eq. (5)) and that these necessary rules are in conflict with the usual Cauchy principal-value integration procedure. We have presented here an alternate form for the angular Green's function, and associated closure condition, which is not burdened by these added, and somewhat confusing, rules.

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⁵N. I. MUSKHELISHVILI, *Singular Integral Equations*, Noordhoff, Groningen (1953).

A Simple Estimate of the Effects of Resonance Interference*

The accurate computation of capture in resonances shows that when resonances occur close together there may be a sizeable effect on the capture rate because of flux perturbations^{1,2}. While an accurate computation is a formidable problem, there are some conditions which a) occur reasonably often, and b) admit a simple approximate answer.

Suppose there are two resonances, labeled I and II, close together. Further assume that by reason

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¹C. N. KELBER, "Fluxes and Reaction Rates in the Presence of Interfering Resonances," *Trans. Am. Nucl. Soc.*, **6**, 2, 273 (1963).

²W. K. FOELL, R. A. GRIMSEY and S. TONG, "A Monte Carlo Study of Resonance Absorption in Gold and Indium Lumps," *Trans. Am. Nucl. Soc.*, **6**, 2, 272 (1963).

of nuclide abundance or level parameters, resonance I dominates the shape of the flux near the resonant energy. Then a simple estimate of the capture rate in resonance II may be obtained by integrating the capture cross section from resonance II times the flux determined by the presence of resonance I in the absence of resonance II.

The procedure is to express the capture cross section in resonance II in terms of the line shape of resonance I and a modifying function. We use unbroadened line shapes and expand σ_{a_2} in the difference $x_1^2 - x_2^2$, where $x_i = 2(E - E_{r_i})/\Gamma_i$; E_{r_i} is the resonance energy and Γ_i is the total width.

For the flux we use the intermediate representation of Goldstein and Cohen³ and introduce their parameter β_λ given by

$$\beta_\lambda^2 = 1 + \frac{\sigma_0}{s + \lambda\sigma_p} \frac{\Gamma_\gamma + \lambda\Gamma_n}{\Gamma},$$

where

the resonance parameters are those of the dominant resonance (I)

σ_0 is the peak cross section of resonance I,

s is the effective moderator scattering per atom of type I

σ_p is the total potential scattering per atom of type I

λ is the intermediate representation parameter.

Performing the integration by a contour integral^a

$$RI_2 = \frac{\Gamma_1}{2E_{r_1}} \int_{-\infty}^{\infty} \psi_1 \sigma_{a_2} dx_1,$$

where RI_2 is the resonance integral for the second resonance

$$\psi_1 \approx (1 + x_1^2)/(\beta_\lambda + x_1^2), \quad \sigma_{a_2} = \frac{\Gamma_{\gamma 2}}{\Gamma_2} \frac{\sigma_{0,2}}{1 + x_2^2},$$

we find

$$RI_2 = I_{0,2} \frac{E_2}{E_1} \times \left[1 + \frac{(1 - \beta_\lambda^2)\Gamma_1}{\beta_\lambda} \frac{\Gamma_2 + \beta_\lambda\Gamma_1}{(\Gamma_2 + \beta_\lambda\Gamma_1)^2 + 4(E_2 - E_1)^2} \right],$$

where $I_{0,2}$ is the infinite-dilution resonance integral.

^aSuggested by E. Pennington, ANL. The contour lies in the upper half plane.

³R. GOLDSTEIN and E. R. COHEN, "Theory of Resonance Absorption of Neutrons," *Nucl. Sci. Eng.*, 13, 132-140 (1962).

Consider as an example a mixture of H:U²³⁸ = 1 and estimate the depression in the response of a gold foil from the interference between the 6.68 eV resonance of U²³⁸ and the 4.91 eV resonance in gold. We have for resonance I:

$$E_1 = 6.68 \text{ eV}$$

$$\sigma_0 = 2.192 \times 10^4 \text{ barns}$$

$$\Gamma_1 = 0.0264 \text{ eV}$$

$$\sigma_s = 20 \text{ barns}$$

$$\lambda = 0$$

$$\beta_\lambda^2 = 954.$$

For resonance II:

$$E_2 = 4.91 \text{ eV}$$

$$\Gamma_2 = 0.1406 \text{ eV}.$$

By substitution we find $\frac{RI_2}{I_{0,2}} = 0.69$; the largest part of the ratio, 0.734, comes from the term E_2/E_1 . It may be fairly argued that the ratio E_2/E_1 should be omitted, since it comes from neglecting the $1/E$ dependence of the flux. In fact, if we put $\beta_\lambda = 1$ (no resonance flux depression) we find $RI_2/I_{0,2}$ given by: $RI_2/I_{0,2} = E_2/E_1$. We therefore suggest modifying the formula given above by dropping the ratio E_2/E_1 . When this is done for the example cited, $RI_2/I_{0,2} = 0.94$. An exact calculation¹ gives a value of 0.92 for this ratio.

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Resonance Integrals for U²³³ Fission and Th²³² Capture

The resonance integrals for U²³³ fission and Th²³² capture have been measured relative to the resonance capture integral of Au¹⁹⁷ by means of the cadmium-ratio-activation technique¹. Dilute detector foils were irradiated in an 11.5 cm diam water hole at the center of the TRX critical facility¹. The TRX is a water-moderated lattice of cylindrical, slightly enriched uranium metal and UO₂ fuel rods. The epithermal flux spectrum in the water hole was approximately proportional to $1/E$ except for the flux peak above 25 keV.

Figure 1 shows the disc-shaped cadmium box.

¹J. HARDY, Jr., D. KLEIN and G. G. SMITH, *Nucl. Sci. Eng.*, 9, 341-345 (1961).