where C_{i0} is the initial delayed-emitter concentration and Q characterizes the external neutron source. The other symbols have their usual meaning.

This reactivity function has been used[2] in defining the equivalent transfer functions of a reactor for both the critical state and the sub- and supercritical state in dependence on the value α. The nonlinear effects (the influence of the amplitude A) and the conditions for using the above-mentioned transfer functions have been also evaluated.

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[1]R. L. MURRAY, C. R. BINGHAM and Ch. F. MARTIN, "Reactor Kinetics Analysis by an Inverse Method," *Nucl. Sci. Eng.*, **18**, 481-490 (1964).

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On the Green's Function of Monoenergetic Neutron Transport Theory*

In recent years several authors (Refs. 1, 2, 3 among others) have used the normal mode approach to the solution of the monoenergetic neutron transport equation. Each author has presented a development of the angular Green's function. We shall illuminate here several misleading aspects which are generated by these previous discussions.

For the sake of brevity, let us consider the case of isotropic scattering in a medium with plane symmetry. In the notation of Ref. 2, the neutron flux resulting from the 'monodirectional' source $\delta(\mu - \mu_0)\,\delta(x)$ is given by

$$\rho_I(x) = \frac{\phi(L,\mu_0)}{M_+} e^{-x/L} + \int_0^{+1} \frac{\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu}\, d\nu, \quad x > 0$$
$$= -\frac{\phi(-L,\mu_0)}{M_-} e^{x/L} - \int_{-1}^{0} \frac{\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu}\, d\nu, \quad x < 0. \tag{1}$$

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[1]K. M. CASE, *Ann. Phys.*, **9**, 1-23 (1960).
[2]J. MIKA, *Nucl. Sci. Eng.*, **11**, 415-427 (1961).
[3]J. J. McINERNEY, *Nucl. Sci. Eng.*, **16**, 460-462 (1963).

It should be noted that $\rho_I(x)$ has no unusual functional properties since the uncollided neutron contribution does not appear as a δ distribution. Thus, it is not surprising that Eq. (1) is consistent with previously published results based on a normal mode expansion. However, Eq. (1) can be derived using the elementary methods of Ref. 4.

Using Eq. (1) we can determine the angular Green's function (i.e. the angular flux resulting from the present source) via the relation

$$\Psi_G(x,\mu;\mu_0) = \Psi_0(x,\mu;\mu_0) + \frac{c}{2}\int_{-\infty}^{+\infty} \frac{e^{-|x-x'|/|\mu|}}{|\mu|}\,\rho_I(x')dx', \tag{2}$$

where $\Psi_0(x,\mu;\mu_0)$ is the uncollided angular flux and c is the scattering probability. It is a straightforward matter to reduce Eq. (2) to the form

$$\Psi_G(x,\mu;\mu_0) = \frac{\phi(L,\mu)\phi(L,\mu_0)}{M_+} e^{-x/L} + \int_0^{+1} \frac{\phi(\nu,\mu)\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu}\, d\nu + h(\mu)\Delta(\mu,\mu_0)e^{-x/\mu}, \quad x > 0 \tag{3}$$
$$= -\frac{\phi(-L,\mu)\phi(-L,\mu_0)}{M_-} e^{x/L} - \int_{-1}^{0} \frac{\phi(\nu,\mu)\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu}\, d\nu - h(-\mu)\Delta(\mu,\mu_0)e^{-x/\mu}, \quad x < 0,$$

where

$$\Delta(\mu,\mu_0) = \frac{\delta(\mu-\mu_0)}{\mu_0} - \frac{\phi(L,\mu)\phi(L,\mu_0)}{M_+} - \frac{\phi(-L,\mu)\phi(-L,\mu_0)}{M_-} - \int_{-1}^{+1} \frac{\phi(\nu,\mu)\phi(\nu,\mu_0)}{M(\nu)} d\nu$$

and $h(\mu)$ is the unit step function (i.e., $h(\mu) = 0$, $\mu < 0$, and $h(\mu) = 1$, $\mu \geq 0$).

The previously reported angular Green's functions have taken the form of Eq. (3) but with $\Delta(\mu,\mu_0) = 0$. Essentially, these previous developments are based on the closure condition for the function set $\{\phi(\pm L,\mu), \phi(\nu,\mu)\}$,

$$\delta(\mu-\mu') = \frac{\mu\phi(L,\mu)\phi(L,\mu')}{M_+} + \frac{\mu\phi(-L,\mu)\phi(-L,\mu')}{M_-} + \mu\int_{-1}^{+1} \frac{\phi(\nu,\mu)\phi(\nu,\mu')}{M(\nu)}\, d\nu. \tag{4}$$

[4]K. M. CASE, F. deHOFFMANN and G. PLACZEK, *Introduction to the Theory of Neutron Diffusion*, U.S. Government Printing Office (1953).

In Eq. (4), we must add the prescription that doubly Cauchy integrals (which will appear when operating with Eq. (4)) are to be evaluated by interchange of integration order without regard to the dictates of the Bertrand-Poincare' transformation. Thus, in using Eq. (4), one must employ the definition

$$\int_{-1}^{+1} \frac{d\mu}{\nu-\mu} \int_{-1}^{+1} \frac{F(\mu,\nu')}{\nu'-\mu} d\nu' = \int_{-1}^{+1} d\nu' \int_{-1}^{+1} \frac{F(\mu,\nu')}{(\nu-\mu)(\nu'-\mu)} d\mu, \tag{5}$$

where $F(\mu,\nu)$ satisfies a Hölder condition in the interval (-1,+1). Of course, Eq. (5) is in conflict with the Bertrand-Poincare' formula[5]

$$\int_{-1}^{+1} \frac{d\mu}{\nu-\mu} \int_{-1}^{+1} \frac{F(\mu,\nu')}{\nu'-\mu} d\nu' = \int_{-1}^{+1} d\nu' \int_{-1}^{+1} \frac{F(\mu,\nu')}{(\nu-\mu)(\nu'-\mu)} d\mu + \pi^2 F(\nu,\nu). \tag{6}$$

Using the closure condition of Eq. (4) in Eq. (3) we note that $\Delta(\mu,\mu_0) = 0$ and thus previous results are in agreement with Eq. (3). We also note that the closure condition is intimately linked with the term representing the uncollided flux. This is to be expected since the unusual functional properties of the angular Green's function are found in the uncollided term.

It is not difficult to find a closure condition for the function set $\{\phi(\pm L, \mu), \phi(\nu,\mu)\}$ that satisfies the 'ordinary' rules of integration as expressed in Eq. (6). The result is

$$\frac{\mu^2\lambda^2(\mu)}{M(\mu)}\delta(\mu-\mu') = \frac{\mu\mu'\phi(L,\mu)\phi(L,\mu')}{M_+} + \frac{\mu\mu'\phi(-L,\mu)\phi(-L,\mu')}{M_-} + \mu\mu' \int_{-1}^{+1} \frac{\phi(\nu,\mu)\phi(\nu,\mu')}{M(\nu)} d\nu, \tag{7}$$

where $\lambda(\mu)$ is given in Ref. 2. Using Eq. (7) in Eq. (3) yields the angular Green's function

$$\Psi_G(x,\mu;\mu_0) = \frac{\phi(L,\mu)\phi(L,\mu_0)}{M_+} e^{-x/L} + \int_0^{+1} \frac{\phi(\nu,\mu)\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu} d\nu + h(\mu)\delta(\mu-\mu_0)\left(\frac{\pi c\mu}{2}\right)^2 \frac{e^{-x/\mu}}{M(\mu)}, \quad x > 0$$

$$= -\frac{\phi(-L,\mu)\phi(-L,\mu_0)}{M_-} e^{x/L} - \int_{-1}^{0} \frac{\phi(\nu,\mu)\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu} d\nu - h(-\mu)\delta(\mu-\mu_0)\left(\frac{\pi c\mu}{2}\right)^2 \frac{e^{-x/\mu}}{M(\mu)}, \quad x < 0. \tag{8}$$

With this functional we need not include any added prescriptions such as the rule of Eq. (5).

Let us also point out that one can approach the problem of determining the angular Green's function by considering a distributed source of the form $S(\mu)\delta(x)$, where $S(\mu)$ satisfies a Hölder condition in the interval (-1,+1). The solution is put in the form

$$\Psi(x,\mu) = \int_{-1}^{+1} S(\mu_0)\Psi_G(x,\mu;\mu_0) d\mu_0. \tag{9}$$

If the rules of integrating Cauchy singular functions are followed (esp. Eq. (6)), then the Green's function which results is that given by Eq. (8).

In conclusion, we note that the angular Green's functions which appear in the literature require a further prescription (as given in Eq. (5)) and that these necessary rules are in conflict with the usual Cauchy principal-value integration procedure. We have presented here an alternate form for the angular Green's function, and associated closure condition, which is not burdened by these added, and somewhat confusing, rules.

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A Simple Estimate of the Effects of Resonance Interference*

The accurate computation of capture in resonances shows that when resonances occur close together there may be a sizeable effect on the capture rate because of flux perturbations[1,2]. While an accurate computation is a formidable problem, there are some conditions which a) occur reasonably often, and b) admit a simple approximate answer.

Suppose there are two resonances, labeled I and II, close together. Further assume that by reason

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[1] C. N. KELBER, "Fluxes and Reaction Rates in the Presence of Interferring Resonances," *Trans. Am. Nucl. Soc.*, **6**, 2, 273 (1963).
[2] W. K. FOELL, R. A. GRIMESEY and S. TONG, "A Monte Carlo Study of Resonance Absorption in Gold and Indium Lumps," *Trans. Am. Nucl. Soc.*, **6**, 2, 272 (1963).