Letters to the Editor

Second-Order Perturbation Theory

In the paper by Greenspan et al.,[1] I do not feel that it is sufficiently emphasized that the optimization algorithm based on the second-order perturbation theory (SOPT) formulation is not a true second-order perturbation. As stated by the authors, for most practical problems a true second-order approach is prohibitively expensive, involving as it does $(I-1)\cdot M$ extra transport calculations at each iteration, where I is the number of materials and M is the number of zones (unless the problem is essentially one-dimensional as, for example, in Ref. 2). If this could be done, however, it would give a distribution of the I materials throughout the shield at each iteration different from that obtained by the method of Greenspan et al.

In the case $I = 2$, Greenspan et al. obtain the optimum shape of the material density change *to first-order*. This is then treated as a perturbation whose optimum size may be found using SOPT. First-order perturbation theory is therefore used to determine the nature of the perturbation and SOPT is subsequently used to determine the size. (I use the Greenspan et al. terminology, where "second-order perturbation" means the retention of terms up to and including second-order derivatives in the Taylor series expansion.) The perturbation determined using the optimum first-order density changes is the best perturbation that can be found with only two transport calculations using ANISN or DOT.

In the case $I > 2$, there are $(I-1)$ competing perturbations produced from the first-order treatment. By use of SOPT, these are combined as in Eq. (50) of Ref. 1 to give an optimum system perturbation. As illustrated in the example of the optimization of the iron-water shield, the Greenspan et al. algorithm produces good convergence and seems to be a very useful tool in this type of problem.

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[1]E. GREENSPAN, D. GILAI, and E. M. OBLOW, *Nucl. Sci. Eng.*, **68**, 1 (1978).
[2]E. T. CHENG, *Trans. Am. Nucl. Soc.*, **27**, 802 (1977).

Reply to a Comment on "Second-Order Perturbation Theory"

The second-order perturbation theory (SOPT) formulation presented in Ref. 1 is a *true* second-order perturbation theory. Burn's comment[2] refers to the specific example brought forth illustrating the application and usefulness of the SOPT.

Generally speaking, it is up to the user to decide whether to apply it using the maximum possible number of control variables (such as the density of each material at each zone) and pay accordingly for the computer time, or to apply it using a reduced number of variables. In our particular example, we determined the shape of the density change guided by first-order perturbation theory and the amplitude of this change using SOPT. This particular combination was found to be most efficient for optimization problems of the type considered. Burn is right in observing that had we used all the $(I-1)\cdot M$ control variables in the SOPT formulation, the material distribution in the intermediate steps, marching from the reference system to the optimal one, would be different from what we obtained in the example. The optimal materials distribution (the identification of which is, after all, the ultimate goal for the optimization work), however, is unique, independent of the intermediate steps.

Our statement[1] that "for most practical problems such an approach is prohibitively expensive" addressed the particular application of SOPT for the optimization of the material distribution in problems having a large number of variables. It is not to be interpreted as applied, in general, to SOPT. Another recent example for the application of SOPT can be found in Ref. 3.

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[1]E. GREENSPAN, D. GILAI, and E. M. OBLOW, *Nucl. Sci. Eng.*, **68**, 1 (1978).
[2]K. W. BURN, *Nucl. Sci. Eng.*, **70**, 215 (1979).
[3]E. GREENSPAN, Y. KARNI, and D. GILAI, "High Order Effects in Cross Section Sensitivity Analysis," *Proc. Sem./Workshop Theory and Application of Sensitivity and Uncertainty Analysis*, Oak Ridge, Tennessee, August 22-24, 1978, ORNL/RSIC-42, Oak Ridge National Laboratory (1978).