which gives two values for the core half-thickness, $h_1 > h_2$, if the reflector half-thickness t is given ($0 < t \leqslant \infty$, with distances measured in units of moderator diffusion length L_M). This means that for identical burnup distributions, two half-thicknesses of the reactor core, i.e., "double criticality," occur.

We first consider the physical interpretation of the phenomenon with the case of infinite reflector. On the core-reflector interface, the burnup attains the value $s(h_2)$. Extending the core into the reflector by uniformly adding fuel with burnup following the prescribed distribution into the region $(h_1 - h_2)$, two trends compete: the increase of reactivity due to the larger core thickness and its decrease due to the increasing burnup of the added fuel. At the beginning of the procedure the former effect prevails, but finally the second becomes more effective. The two effects cancel at core half-thickness h_1, making the larger core critical. It is evident that a region with negative buckling will accrue in the core.

It can be perhaps claimed that in the case described, a part (with half-thickness h_2) of a critical reactor (with half-thickness h_1) is also critical.

In the reactor with a finite reflector two cases could be distinguished as follows.

1. Extracting the fuel and moderator from the region $h_1 - h_2$ and shifting the reflector to the core, the resulting smaller reactor will also maximize the average burnup and h_2 will be given by the root of Eq. (2).

2. Extracting only fuel from the region $h_1 - h_2$ also changes the value of h_2 because this also affects the reflector thickness, changing it to the value $h_1 - h_2 + t$. In this case, a part of the larger reactor will be critical as well, but the (unchanged) burnup distribution in the core will not yield the maximal average burnup of the fuel.

Consequently it seems to be certain that in a reactor with nonuniform distribution of physical parameters (multiplication, absorption, etc.) a "strange" behavior (e.g., double criticality) may occur, probably due to the fact that the region with negative buckling is present in the core.

I wonder whether the double criticality described in Ref. 1 has any physical explanation, or whether it is merely the consequence of the mathematical model used; namely, that of two-group theory.

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July 1, 1980

Reply to "On 'Double Criticality' "

During the preparation of our earlier communication,[1] we were surprised there were no references to "double criticality" and apologize to Bartosek for not having been aware of his publications.

In response to his Letter,[2] we point out that the problems discussed in Refs. 1 and 2 are slightly different.

If a symmetric slab reactor is parted at the center and if the space is filled with a material whose k_∞ is unity, the new system is critical for any size of the central region. In our problem, the imposed conditions, such as the continuity of the fuel density, happen to be met at one particular point.

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[1]Y. ISHIGURO et al., *Trans. Am. Nucl. Soc.*, **33**, 372 (1979).
[2]VACLAV BARTOSEK, *Nucl. Sci. Eng.*, **78**, 104 (1981).

Editor's Note: Although the above exchange stems from a communication not in *Nuclear Science and Engineering*, thereby not meeting one of the usual criteria of Letters, we believe the Society has a responsibility to provide a forum for comment no matter which Society publication is involved. Until more appropriate outlets develop, *Nuclear Science and Engineering* will consider providing, upon request, such a forum.

Neutron Lifetime, Generation Time, and Reproduction Time

Marotta has courteously shown me an advance copy of his work employing the concept of the excess time.[1] I am no Monte Carlo expert and cannot usefully comment on the application in this area. But I feel I am responsible for a certain amount of confusion in giving the name "generation time" some years[2] ago to a concept pioneered by Henry.[3] May I make belated amends? I would now prefer the name "reproduction time"[4] and distinguish this from what Hurwitz[5] called the generation time. I hope the following explanation with values in the simplest model of a reactor will make the distinction:

l = neutron lifetime

= mean time for one neutron to be removed from the reactor

$$= \frac{1}{(\Sigma_a + DB^2)v}$$

Λ = neutron reproduction time

= mean time for one neutron to be replaced by another neutron on fissioning

$$= \frac{1}{\nu\Sigma_f v}$$

τ = neutron generation time

= mean time for one neutron to cause fission, i.e., to bring about the next generation

$$= \frac{1}{\Sigma_f v} \ .$$

It is well known[4] that what is "production" and what is "removal" [e.g., $(n, 2n)$ processes] is something of an arbitrary definition, but within such limits, one can say

[1]C. R. MAROTTA, *Nucl. Sci. Eng.*, **77**, 107 (1981).
[2]J. LEWINS, *Nucl. Sci. Eng.*, **7**, 122 (1960).
[3]A. F. HENRY, *Nucl. Sci. Eng.*, **3**, 52 (1958).
[4]J. LEWINS, *Nuclear Reactor Kinetics and Control*, Pergamon Press Ltd., Oxford, United Kingdom (1978).
[5]H. HURWITZ, Jr., *Nucleonics*, **5**, 61 (July 1949).

$$l = \frac{1}{\text{removal rate}}$$

and

$$\Lambda = \frac{1}{\text{production rate}}$$

and define

$$k_{\text{eff}} = \frac{\text{production rate}}{\text{removal rate}}$$

and

$$\rho = \frac{\text{production rate} - \text{removal rate}}{\text{production rate}} \ .$$

Then we have

$$\rho = \left(\frac{1}{\Lambda} - \frac{1}{l}\right)\bigg/\frac{1}{\Lambda} = (l - \Lambda)/l \ .$$

In a *critical* reactor, $\rho = 0$ and $l = \Lambda$ or $l = \tau/\nu$.

The maximum reactivity possible (which relates perhaps to Marotta's concept of efficient utilization) will come when all neutron losses are zero. Putting $\Sigma_a = \Sigma_f + \Sigma_c$, "losses" are due to $\Sigma_c + DB^2$:

$$\rho = \frac{(\nu - 1)\Sigma_f - (\Sigma_c + DB^2)}{\nu\Sigma_f} \ ; \ \rho_{\max} \to \frac{\nu - 1}{\nu} \ .$$

Correspondingly,

$$\Lambda = \frac{1}{\nu\Sigma_f v} \ ; \ l_{\max} \to \frac{1}{\Sigma_f v} \ ; \ \tau_{\max} \to \frac{1}{\Sigma_f v} \ .$$

Thus, $l - \tau \to 0$ at the physical maximum and $l - \Lambda \to l(\nu - 1)/\nu$. In the Monte Carlo sense, τ but not Λ is a subset of l.

It is not clear to me whether Marotta's g is my τ or my Λ. I would also comment that the choice of weighting function in Monte Carlo calculations, like other calculations of static eigenvalues, introduces a somewhat arbitrary linear scaling of ρ (or k_{eff}) (Ref. 6) when the properties are not uniform in space and velocity as supposed in the simple model used here.

I hope the above simple account of the concepts helps clarify misunderstandings to which I may have contributed.

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[6]A. M. WEINBERG and E. P. WIGNER, *The Physical Theory of Neutron Chain Reactors*, Chicago (1958).

Response to "Neutron Lifetime, Generation Time, and Reproduction Time"

I thank Lewins for clarifying in Ref. 1 some ambiguous basic ideas and nomenclature in this area. I would like to compare the definitions of g and l as I used them in Ref. 2 and their relationship to Λ and τ as used by Lewins in Ref. 1. I follow Lewins' format in the comparison Table I.

The KENO generation time g is calculated by multiplying the neutron elapsed time by the ν-fission probability and thus is equivalent to Lewins' Λ. Unfortunately, different names are used for the same concept. Lewins' suggested reproduction time should be adopted—especially since generation time (as introduced by Hurwitz) has historic precedence.

I agree with Lewins' formulation in Ref. 1 that in a critical reactor one should expect l and g (= Λ) to be equal. I also agree that, at the physical maximum state of k_{eff}, l should equal $\bar{\nu}g$ (= τ) where $\bar{\nu}$ is the average number of fast neutrons produced per fission.

The above equalities are at variance with my results of Ref. 2. There it was established, for a complex coupled fissionable system, that l and g (= Λ) are equal only at the point of maximum utilization of neutrons (excess time $E = l$ and $g = 0$), which happened to be also the point of maximum k_{eff} of the configuration. The maximum slope of k_{eff} indicates the point of maximum neutron utilization ($E = 0$, $l = g$) of the system. In this calculation, it was also shown that two critical states exist for the system; both, however, indicate that l and g are not equal for k_{eff} of unity. It would therefore appear that the elementary theory of Ref. 1 cannot account for complex interaction and therefore cannot be used for reliable guidance for reactivity values or trends.

Two uncoupled simple systems have been analyzed using the same methodology as in Ref. 2 to explore further the l and g relationship. I calculated k_{eff} versus moderator (water) density for one of the 200 fuel assemblies comprising the array of Ref. 2. This was a 17 × 17 U(3)O_2-rod[3] light water reactor fuel assembly submerged in water with a 1-ft-thick all-around water reflector. The pertinent KENO results are given in Table II and Fig. 1, where k_{eff} and E have been added as dashed curves to Fig. 1 of Ref. 2. We note that at full moderator density, k_{eff} of the assembly agrees well with that of the array since all the 200 assemblies of the 20 × 10 array

TABLE I

Parameter	Designation by	
	KENO[a]	Lewins[b]
Neutron lifetime[c]	l	l
Neutron reproduction time[d]	g (called generation time in KENO)	Λ
Generation time[e] (Hurwitz)	Not used	τ

[a]As used in Ref. 2.
[b]As used in Ref. 1.
[c]The average life span of a neutron until it escapes or gets absorbed.
[d]The average time it takes a neutron to produce another neutron.
[e]The average time taken by a neutron to cause a fission in a steady-state fission distribution. See Ref. 4.

[1]J. D. LEWINS, *Nucl. Sci. Eng.*, **78**, 105 (1981).
[2]C. R. MAROTTA, *Nucl. Sci. Eng.*, **77**, 107 (1981).

[3]Here U(3) denotes uranium enriched to 3% in the ^{235}U isotope.
[4]H. HURWITZ, Jr., *Nucleonics*, **5**, 61 (July 1949).