Letters to the Editor

Comments on "Theoretical and Experimental Criteria for Nonlinear Reactor Stability"

In a recent paper, Gyftopoulos[1] derived a frequency domain criterion which, if satisfied, was to ensure the asymptotic stability of a system describable by the point reactor kinetics equations, all delayed neutron precursors, and a linear or nonlinear feedback. Asymptotic stability was concluded on the premise that Lyapunov's asymptotic stability theorem was satisfied. It is shown below that the V function derived by Gyftopoulos is not positive definite, hence not a Lyapunov function. Thus, Lyapunov's asymptotic stability theorem was not satisfied and the resulting frequency domain criterion lacks proper foundation.

The V function and the properties assigned to it by Gyftopoulos are

$$V = p(t) - \ln[1+p(t)] - \frac{1}{2d^2}p^2(t)$$
$$+\sum_i^m \frac{\beta_i}{\lambda_i\Lambda}\left\{c_i(t) - \ln[1+c_i(t)] - \frac{1}{2d^2}c_i^2(t)\right\}$$
$$+\sum_i^m \frac{\beta_i}{\Lambda}\int_{-\infty}^t [p(\tau)-c_i(\tau)]^2\left\{\frac{1}{[1+p(\tau)][1+c_i(\tau)]} - \frac{1}{d^2}\right\}d\tau$$
$$+\frac{b\Lambda}{a}\int_{-\infty}^t \frac{a-1-p(\tau)}{1+p(\tau)}k^2(\tau)d\tau ,$$

where a, b, and d are positive numbers with a, $d > 1$.

If $d \geq a$; $-1 < p < a-1$; $-1 < c_i < a-1$, $i = 1, 2, \ldots m$.

If $d < a$; $-1 < p < d-1$; $-1 < c_i < d-1$, $i = 1, 2, \ldots m$.

V is zero when $p = c_i = 0$ and the two integrals equal zero, otherwise it is positive.

Recall from the definition of positive definiteness[2] that $V(x) > 0$ for $x \neq 0$ and $V(0) = 0$, where x is the state vector. Assuming the system to be asymptotically stable, consider the behavior of the Gyftopoulos V function due to a disturbance from equilibrium. Prior to the disturbance $p = c_i = 0$, the two integrals are zero, hence $V = 0$. Following the disturbance, the integrals take on and retain a positive value for all future time even after the state vector has decreased to zero. This contradicts the above properties, which are part of the definition of positive definiteness. Q.E.D.

Sufficiency of Gyftopoulos's frequency domain criterion or some modification of it might yet be shown by redefining the V function such as to satisfy all the properties required by Lyapunov's theory. Alternately, it might be possible to reestablish the analytical basis for the frequency domain criterion by showing asymptotic stability in some non-Lyapunov sense.

Joseph E. Kalinowski

Department of Nuclear Engineering
Purdue University
Lafayette, Indiana 47907

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[1]ELIAS P. GYFTOPOULOS, *Nucl. Sci. Eng.*, **26**, 26 (1966).

[2]LaSALLE and LEFSCHETZ, *Stability by Liapunov's Direct Method with Applications*, p. 33, Academic Press, New York (1964).

Reply to "Comments on 'Theoretical and Experimental Criteria for Nonlinear Reactor Stability'"

Mr. Kalinowski's comments are not applicable to Ref. 1. The reason is stated in the first paragraph, on p. 28 of the reference. The statement is as follows: "Note that, since the system of Eqs. (1)-(3) is autonomous, the function V is not explicitly dependent on time. It is convenient, however, to write the two integrals in Eq. (17) as functions of t to avoid the necessity for the definition of new variables and the consideration of partial derivatives with respect to these variables."

In other words, each of the integrals in Eq. (17) must be regarded as representing a positive definite function $L_i(z)$ of one or more auxiliary variables. This function must be such that: a) for $z = 0$, $L_i(0) = 0$, and $dz/dt = 0$; and b) dL_i/dt equals the integrand of the corresponding integrand of the corresponding integral, evaluated at $\tau = t$.

Thus, the contradiction discussed by Mr. Kalinowski does not arise.

Elias P. Gyftopoulos

Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

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[1]ELIAS P. GYFTOPOULOS, *Nucl. Sci. Eng.*, **26**, 26 (1966).