Letter to the Editor

On Rafalski's Solution to the Albedo Problem

In an article in this journal, Rafalski[1] has considered the monoenergetic transport problem of computing the probability that a neutron will be reflected if it is perpendicularly incident upon a semi-infinite halfspace. The halfspace is assumed homogeneous with isotropic scattering. Rafalski's method of solution is to approximate the solution of the integral transport equation by partially assuming the flux to be in an asymptotic distribution. He found that the resulting simple expression for the albedo was quite accurate for highly absorbing systems, but that the accuracy became worse as the system became less absorbing. This is a somewhat unusual result since generally any approximation based on the assumption of an asymptotic distribution for the neutron flux is most accurate for weakly absorbing systems.

In this note we would like to point out the interesting fact that Rafalski's approximate solution to the normal beam albedo problem is, in fact, an exact transport solution to the albedo problem for another simple incident distribution. This observation leads to a very simple explanation as to why Rafalski's results are most accurate for highly absorbing systems. Consider an incident neutron flux of the form

$$\varphi(z,\mu)|_{z=0} = (1 - \mu\nu)^{-1} \quad , \quad \mu > 0 \quad , \tag{1}$$

where ν is the positive root of

$$\frac{2\nu}{c} = \ln\left(\frac{1+\nu}{1-\nu}\right) \quad , \tag{2}$$

with c representing the mean number of secondary neutrons per collision. The remaining notation is standard. The exact transport solution for the flux in the halfspace is then given by

$$\varphi(z,\mu) = (1 - \mu\nu)^{-1} \exp(-\nu z) \quad . \tag{3}$$

Equation (3) is obviously the solution for the flux since it satisfies both the integro-differential transport equation and the boundary condition, and the solution to the transport equation is known to be unique.[2] The probability of reflection, i.e., the albedo A, is then given by

$$A = \frac{J_{\text{out}}}{J_{\text{in}}} = \frac{\int_{-1}^{0} d\mu\,|\mu|\,(1-\mu\nu)^{-1}}{\int_{0}^{1} d\mu\,\mu\,(1-\mu\nu)^{-1}} = \frac{\ln(1+\nu) - \nu}{\ln(1-\nu) + \nu} \quad . \tag{4}$$

Equation (4) is an exact transport result for the incident distribution given by Eq. (1).

Rafalski's approximate solution to the normal beam problem is[1]

$$A(\text{Rafalski}) = 1 - (1-c)/\{1 - c[\nu + \ln(1+\nu)]/2\nu\} \quad . \tag{5}$$

Use of Eq. (2) to eliminate c in Eq. (5) yields

$$A(\text{Rafalski}) = \ln(1+\nu) - \nu/[\ln(1-\nu) + \nu] \quad . \tag{6}$$

Comparison of Eqs. (4) and (6) shows that Rafalski's approximation to the normal beam problem is identical to the exact solution corresponding to an incident distribution given by Eq. (1). Having established this identity, it is clear why Eq. (6) is most accurate for highly absorbing systems ($c \approx 0$). For small c, the parameter ν is very close to unity and the distribution given by Eq. (1) is very peaked near $\mu = 1$, i.e., it appears almost as a normal beam. Hence, for small values of c the problem for which Eq. (6) is an exact result is very nearly the normal beam problem. On the other hand, for a highly scattering system ($c \approx 1$) the parameter ν is almost zero and Eq. (1) represents an almost isotropic incident distribution. In this case one would expect Rafalski's result to be a relatively poor approximation to the normal beam albedo. However, it should give a good approximation for the albedo corresponding to an isotropic incident distribution. This is in fact the case. For $c = 0.99$, Eq. (6) gives $1\text{-}A = 0.207$ whereas the exact solution[1] to the normal beam problem is $1\text{-}A = 0.247$; however, the exact solution for an isotropic incident flux[3] is $(1\text{-}A) = 0.205$ which agrees very well with Rafalski's result. A simple expression for the albedo which is accurate for both the normal beam and isotropic incidence problems for all values of c is available in the literature.[4]

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[1] P. RAFALSKI, *Nucl. Sci. Eng.*, **19**, 378 (1964).
[2] K. M. CASE and P. F. ZWEIFEL, *Linear Transport Theory*, Addison-Wesley, London (1967).
[3] D. S. SELENGUT, private communication (July 1963).
[4] G. C. POMRANING, *Nucl. Sci. Eng.*, **21**, 265 (1965).