NUCLEAR SCIENCE AND ENGINEERING: 25, 450-451 (1966)

Letters to the Editor

The Decay Constant of a Neutron Pulse*

There has been considerable interest, recently[1-3], in the question of time decay constants in small, crystalline systems. Specifically, one is puzzled by the unequivocal observation of "fundamental" decay constants which transcend the critical value $\lambda^* \equiv (v\Sigma)_{min}$. In a recent note, one of us (N. C.) averred that the phenomenon could surely be understood through a study of the continuum contribution in the eigenfunction expansion of the pulse, $n(v, t)$. We wish to report, here, the results of an exact calculation, which bear out this point of view.

We have computed the time-dependent activation of a $1/v$ detector located in the moderating sample. The simplified Boltzmann equation which we use has the following features:

a) leakage is described by diffusion theory, throughout,

b) scattering is described by a kernel composed of an elastic, isotropic part and an inelastic, isotropic part written as a one-term degenerate kernel. Thus,

$$\Sigma_s(v',v) = \Sigma_e(v)\,\delta\,(v'-v) + \beta\Sigma_i(v')v\Sigma_i(v)M\,(v), \quad (1)$$

with

$$\beta^{-1} = \int_0^\infty dv\; v\,\Sigma_i(v)M\,(v). \quad (2)$$

Σ_e and Σ_i are displayed in Fig. 1.

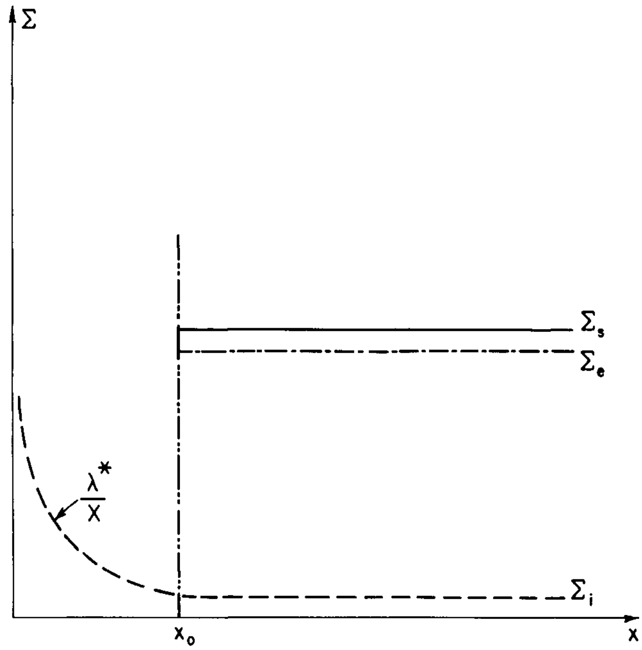

Fig. 1. Cross sections for elastic and inelastic scattering. $x^2 = E/kT$.

The model, while schematic in nature, has been designed to include the important physical features of the problem. The diffusion term, while not quite correct in small systems, does provide a loss mechanism that varies inversely with surface area, and the scattering kernel does exhibit a Bragg cut-off. The calculation of $n(v, t)$ becomes a problem of relatively simple quadrature, and the behavior of λ_0 as a function of B^2, which is of primary interest, has perhaps semi-quantitative validity.

Straightforward calculation, in the manner sketched in Ref. (4), permits us to express the detector response as

$$N(t) = N_0 \exp(-\lambda_0 t) + \int_{\lambda *}^\infty d\lambda\; \exp(-\lambda t)N(\lambda). \quad (3)$$

When $0 < B^2 < B^2_{cr}$, $N(\lambda)$ is an irregular function of λ. It has two maxima, at values of λ which depend upon B^2, and which are shown by the broken curves in Fig. 2. The peaks owe their existence to the singular behavior of cross sections at the Bragg energy.

When $B^2 > B^2_{cr}$, the first term in (3) is absent and $(N\lambda)$ contains first two peaks, then, only one. The first peak is extremely tall and narrow $[(\Delta\lambda/\lambda) \sim (1/75)]$ and depends upon B^2 in a manner that suggests it be considered the the natural continuation of $\lambda_0(B^2)$. It is so displayed in Fig. 2. The strong peak dominates the decay for a considerable interval, but eventually gives way to the "slower" portions of the continuum contribution. In our model, for example,

$$N(t) \approx \exp(-\lambda_p t) + 0.12\,\frac{\exp(-\lambda^* t)}{(\lambda^* t)^{3/2}} \quad \lambda^* t > 1 \quad (4)$$

when $B^2 = 1.2\, B^2_{cr}$, $\lambda_p = 1.2\,\lambda^*$. Here, one would have to wait until $\lambda^* t \gtrsim 30$ before the asymptotic, non-exponential decay became significant. The "waiting-time" in this case is ~ 10 msec.

The peak, λ_p, may be characterized mathematically in the following way[5]. The critical equation for λ_0 may be

*This work was performed under the auspices of the USAEC.

[1]See, for example, *Proc. IAEA Symp. Pulsed Neutron Research*, Karlsruhe (May 1965).

[2]I. C. GOYAL and L. S. KOTHARI, *Nucl. Sci. Eng.*, **23**, 159 (1965); L. S. KOTHARI, *ibid.*, **23**, 402 (1965).

[3]N. CORNGOLD, *Nucl. Sci. Eng.*, **23**, 403 (1965); M. M. R. WILLIAMS, *J. Nucl. Energy* (to be published); J. WOOD (unpublished research).

[4]N. CORNGOLD, P. MICHAEL, W. WOLLMAN, *Nucl. Sci. Eng.*, **15**, 13 (1963), Appendix B.

[5]A related approach has been used in the kinetic theory of gases by L. Sirovich and J. Thurber. See, for example, *Proc. Third Intern. Symp. Rarefied Gas Dynamics*, 1962, Vol. I, Acad. Press Inc., New York (1963).

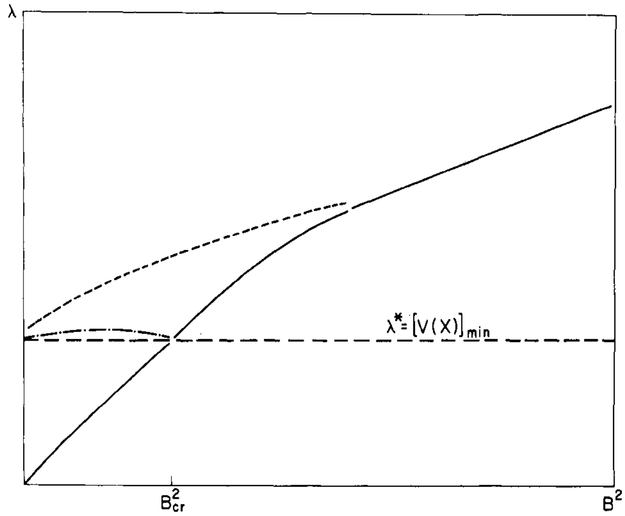

Fig. 2. $\lambda(B^2)$. When $\lambda < \lambda^*$, λ is the discrete decay constant, λ_0. When $\lambda > \lambda^*$, λ is λ_p of text.

written as $\beta B(\lambda) = 1$. (Ref. 4). It has a unique solution, λ_0, when $B^2 < B^2_{cr}$. We may obtain a solution when $B^2 > B^2_{cr}$ if we regard $B(\lambda)$ as one branch of a multiple-valued function, $\overline{B}(\lambda)$. Then, when $B^2 > B^2_{cr}$, the solution will no longer be found on the principal branch. In our model, λ_0 becomes (at least) a complex pair of solutions, one located on the branch of $\overline{B}(\lambda)$ which lies immediately "above" the principal branch, the other on the branch immediately "below". The contributions from the poles may be extracted by contour deformation, and the response may *always* be written as the sum of "discrete" and "continuous" portions. We have found that the complex pair is quite close to the branch cut. They are responsible for the strong peak in $N(\lambda)$ when the representation of Eq. (3) is used. We have also noted that when Σ_e is set equal to zero the peak, λ_p, in $N(\lambda)$ disappears. In that case,no solutions to $\beta B(\lambda) = 1$ have yet been found on neighboring branches.

Before concluding, one might ask whether $\lambda_p(B^2)$ resembles the function obtained through the cut-off procedure recently suggested by Kothari[2]. We have interpreted the suggestion to mean that we should replace $\beta B(\lambda) = 1$ by $\beta B'(\lambda) = 1$, where B' is computed by integrating over a restricted range of energies. Then, we find that the cut-off produces a discrete λ_0 which differs from λ_p by less than 5% in the range $\lambda^* < \lambda < 2\lambda^*$. The function $\lambda_K(B^2)$, which appears in the cut-off theory as the asymptote of λ_0, also appears in our analysis. We find that, for sufficiently large B^2, λ_p becomes $(v\Sigma_i + vDB^2)_{v_0^+}$. This asymptotic portion appears in Fig. 2 as the final, Linear segment of $\lambda(B^2)$.

The results we have described indicate how "decay constants" might be observed over an unexpectedly large range of bucklings in crystalline moderators. We hope to publish the details of our work, and its obvious extensions, before long.

Noel Corngold and Kanat Durgun†

Brookhaven National Laboratory
Upton, New York

March 24, 1966

†IAEA Fellow on leave from Institute for Nuclear Energy, Technical University of Istanbul.