Letters to the Editor

A Short Table of Indefinite Integrals of the Exponential Integral Function

Problems in radiative transfer frequently result in integrals involving the exponential integral functions defined by

$$E_n(x) = \int_1^\infty \exp(-xt)t^{-n}\,dt \ .$$

However, indefinite integrals involving these functions are seldom included in the standard collections. At various times I have worked out a number of these integrals and believe that this short table will be useful to readers of *Nuclear Science and Engineering*. One or two of the obvious general forms are included for utility.

TABLE I

Indefinite Integrals Involving E_1 and E_2

1.	$\int \frac{E_1(r)}{r^3}\,dr = \frac{E_3(r) - E_1(r)}{2r^2}$
2.	$\int \frac{E_1(r)e^{-r}}{r^2}\,dr = \frac{1}{2}E_1^2(r) + \frac{E_2(2r) - E_1(r)e^{-r}}{r}$
3.	$\int \frac{E_1(r)e^{-r}}{r}\,dr = -\frac{1}{2}E_1^2(r)$
4.	$\int \frac{E_1(r)e^{-r}}{r^3}\,dr = \frac{E_3(2r) - E_1(r)e^{-r}}{2r^2} - \frac{E_2(2r) - E_1(r)e^{-r}}{2r} - \frac{1}{4}E_1^2(r)$
5.	$\int \frac{E_2(r)}{r}\,dr = E_2(r) - E_1(r)$
6.	$\int \frac{E_2(r)e^{-r}}{r^2}\,dr = -\frac{E_1(r)E_2(r)}{r} - \frac{E_2(2r) - E_1(r)e^{-r}}{r} - \frac{1}{2}E_1^2(r)$
7.	$\int \frac{E_1(r)E_2(r)}{r}\,dr = E_1^2(r) + \frac{E_2(2r) + E_1(r)E_2(r) - E_1(r)e^{-r} - E_2^2(r)}{r}$
8.	$\int \frac{E_2(r)e^{-r}}{r}\,dr = E_1^2(r) + \frac{E_2(2r) + E_1(r)E_2(r) - [E_1(r) + E_2(r)]e^{-r}}{r}$
9.	$\int \frac{E_2^2(r)}{r^2}\,dr = -E_1^2(r) - 2\,\frac{E_2(2r) + E_1(r)E_2(r) - E_1(r)e^{-r}}{r} + \frac{E_2^2(r)}{r}$
10.	$\int E_1^2(r)\,dr = -\frac{E_2(2r) - [E_1(r) + E_2(r)]e^{-r} + E_1(r)E_2(r)}{r} - E_1^2(r) - E_1(r)E_2(r)$
11.	$\int rE_1(r)\,dr = \frac{1}{2}r^2E_1(r) - \frac{1}{2}(r-1)e^{-r}$
12.	$\int e^{-r}E_1(r)\,dr = E_1(2r) - E_1(r)e^{-r}$

(Continued)

TABLE I (Continued)

13.	$\int re^{-r}E_1(r)dr = rE_1(2r) - rE_1(r)e^{-r} - E_1(r)e^{-r} + E_1(2r) + \frac{1}{2}E_2(2r)$
14.	$\int rE_1^2(r)dr = \frac{1}{2}\left[r^2E_1^2(r) - 2E_1(r)e^{-r} + \frac{1}{2}e^{-2r} + 2E_1(2r) + rE_1(2r) + \frac{1}{2}E_2(2r)\right]$
15.	$\int E_2^2(r)dr = \frac{1}{3}\left\{rE_2^2(r) - 2rE_1(r)E_2(r) + 2rE_1(r)e^{-r} - 2rE_2(2r) + 2E_1(r)e^{-r} + \frac{1}{2}\exp(-2r) + (r+6)E_1(2r) + \frac{1}{2}E_2(2r) - 2E_2^2(r) - 2E_3(2r)\right\}$
16.	$\int r^nE_1(r)dr = \frac{1}{n+1}\left\{r^{n+1}E_1(r) + \int r^ne^{-r}dr\right\} \quad n = 1,2,3,\ldots$
17.	$\int r^nE_2(r)dr = \frac{1}{n+1}r^{n+1}E_2(r) + \frac{1}{(n+1)(n+2)}\left\{r^{n+2}E_1(r) + \int r^{n+1}e^{-r}dr\right\} \quad n = 1,2,3,\ldots$
18.	$\int \frac{E_1(r)}{r^n}dr = \frac{1}{(n-1)}r^{-(n-1)}\{E_n(r) - E_1(r)\} \quad n = 2,3,4,\ldots$
19.	$\int \frac{E_2(r)}{r}dr = E_2(r) - E_1(r)$
20.	$\int \frac{E_2(r)}{r^n}dr = \frac{1}{n-1}r^{-(n-2)}\left\{\frac{E_n(r) - E_1(r)}{n-2} - rE_2(r)\right\} \quad n = 3,4,5,\ldots$

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June 18, 1986

On the Derivation of the Equations of the Point-Reactor Model

Many authors have derived the equations for the time-dependent behavior (or kinetics) of a point reactor from diffusion and transport theory; see, for example, Refs. 1 and 2. The only aim of this letter is to present a technique of variables separation to obtain the equations of formulation of the point-reactor model from diffusion theory.

In diffusion theory, the system of equations used is (see, for example, Ref. 1)

$$\frac{\partial N}{\partial t} = Dv\nabla^2 N - \Sigma_a vN + (1-\beta)k_\infty \Sigma_a vN + \Sigma_i \lambda_i C_i + S_0 \quad (1)$$

and

$$\frac{\partial C_i}{\partial t} = \beta_i k_\infty \Sigma_a vN - \lambda_i C_i \ , \quad (2)$$

where

$N(\vec{r},t)$ = neutron number density

D, v, Σ_a = diffusion constant, neutron speed, and macroscopic neutron absorption cross section, respectively

$$\beta = \Sigma_i \beta_i \ ,$$

where

β_i = delayed neutron for the i'th emitter

k_∞ = infinite-medium reproduction factor

$\lambda_i, C_i(\vec{r},t)$ = decay constant and density of the i'th type of precursor, respectively

$S_0(\vec{r},t)$ = extraneous neutron source.

All coefficients D, v, Σ_a, k_∞, β, β_i, and λ_i are constant.

We assume that

$$N(\vec{r},t) = g(\vec{r}) + n(t)f(\vec{r}) \ , \quad (3)$$

$$C_i(\vec{r},t) = p_i(\vec{r}) + c_i(t)f(\vec{r}) \ , \quad (4)$$

and

$$S_0(\vec{r},t) = q(t)f(\vec{r}) \ . \quad (5)$$

Substituting Eqs. (3), (4), and (5) into Eqs. (1) and (2) yields

$$\frac{dn}{dt} + \Sigma_a vn - (1-\beta)k_\infty \Sigma_a vn - Dvn\frac{\nabla^2 f}{f} - \Sigma_i \lambda_i c_i - q = Dv\frac{\nabla^2 g}{f} - \frac{\Sigma_a vg}{f} + \frac{(1-\beta)k_\infty \Sigma_a vg}{f} + \frac{\Sigma_i \lambda_i p_i}{f} \quad (6)$$