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Letters to the Editor

Comment on Carlvik's Paper on Dancoff Correction in Lattices

This letter describes a simple FORTRAN routine for the function $Ki_3(x)$ referred to by Carlvik.[1] It has at least the accuracy required.[2] Such a routine might not be conveniently available to many readers.

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              FUNCTIØN BIC3(X)
 BIC3=(.52197980+X*(3.4150861+X*(2.8822975+X*.49523037)))/
1     (.66459895+X*(4.1979495+X*(2.7422822+X*.39516928)))/
2     (SQRT(1.+X)*EXP(X))
 RETURN
 END
```

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[1] I. CARLVIK, *Nucl. Sci. Eng.*, **29**, 325 (1967).
[2] I. GARGANTINI and T. POMENTALE, Comm ACM, **7**, 12, 727 (1964).

On the Connection Between Absorption and the Anisotropy of Scattering in Neutron Transport Theory

In a previous paper[1] the following rather remarkable theorem was proved. Consider one-speed neutron transport in a medium with a scattering law characterized by Legendre components $p_1, p_2 \ldots, p_n$. In other words,

$$f(\mu_0) = \sum_{\ell=0}^{N} \frac{2\ell+1}{4\pi} p_\ell P_\ell(\mu_0) \quad , \tag{1}$$

where μ_0 is the scattering angle and P_ℓ a Legendre polynomial. We denote by c the ratio of scattering-to-absorption in this medium. Then, the diffusion area γ_0^2 can be expressed in a power series in $(1-c)$ of the form

$$\gamma_0^2 = \sum_{m=1}^{N} \alpha_m (1-c)^m \quad . \tag{2}$$

The theorem states that the α_m are the same function only of $p_1, p_2, \ldots, p_m$ for *any scattering law*.

To state it in another way, consider γ_0^2 for a medium described by $p_1, \ldots p_n$, $p_{n+1} = p_{n+2} = p_{n+3} = \ldots = 0$. Consider *another* medium described by $p_1, \ldots, p_n$, $p_{n+1} \neq 0$, $p_{n+2} = p_{n+3} = \ldots = 0$. The theorem states that the first n terms of Eq. (2) are *identical* for the two media.

In a subsequent work[2] the same theorem was shown to apply to the "effective source strength"[3] for the asymptotic solution. Thus, in fact, the theorem also applies to the asymptotic Green's function flux in an infinite medium.

The utility of this theorem is that it permits one to determine immediately the magnitude of the effect a given anisotropy will produce on the asymptotic flux. For example, if it is known that a problem can be described in sufficient accuracy by terms of order $(1-c)^N$, then all anisotropic components of the scattering p_j, $j > N$ can be ignored.

The purpose of this letter is to point out that a similar simplification does not appear to hold for half-space problems. The proof is simply the production of a *Gegenbeispiel*. We consider the expression[4] for the extrapolated endpoint $z_0(c)$. For isotropic scattering it has the form

$$CZ_0(c) = 0.710446[1 - 0.0199(1-c)^2 + \ldots] \quad . \tag{3a}$$

For linearly anisotropic scattering, we have[4]

$$CZ_0(c) = \frac{0.710446}{1-p_1}[1 - 0.0199(1-c)^2 + \ldots] \quad . \tag{3b}$$

Consider the case $c=1$, with linearly anisotropic scattering. Then,

$$Z_0 = \frac{0.710446}{1-p_1} \quad . \tag{4}$$

If the theorem were true for this half-space problem, the value of z_0 for $c=1$ for *quadratically anisotropic scattering* would not depend upon p_2. However, in Case and Zweifel,[4] page 172, z_0 is calculated for quadratically anisotropic scattering, and it is seen that it does indeed depend upon the value of p_2. This proves that the theorem does not apply to the Milne problem and presumably it does not apply to other half-space problems or to problems involving finite media.

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[1] E. İNÖNÜ and A. İ. USSELI, *Nucl. Sci. Eng.*, **23**, 251 (1965).
[2] A. İ. USSELI, Thesis, Middle East Technical University (1967).
[3] A. M. WEINBERG and E. P. WIGNER, *The Physical Theory of Neutron Chain Reactors*, The University of Chicago Press, Chicago, Illinois (1958).
[4] K. M. CASE and P. F. ZWEIFEL, *Linear Transport Theory*, Addison Wesley, Reading, Massachusetts (1967).