and

$$\mathbf{\Omega}\cdot\nabla\phi_\delta = \frac{1}{\delta}\{Q_s[\phi_\delta] + Q_f[\phi_\delta] - \Sigma_t\phi_\delta\}$$
$$= \frac{1}{\delta}Q[\phi_\delta] - \frac{\Sigma_t}{\delta}\phi_\delta \ . \quad (10)$$

The quantities appearing (in abbreviated notation) in the above equations are defined as follows:

1. The quantities $\phi(\mathbf{r},E,\mathbf{\Omega})$ and $\Sigma_t(\mathbf{r},E)$ are the (space-, energy-, and angle-dependent) flux and the (space- and energy-dependent) total macroscopic cross section.

2. The quantity

$$Q_s[\phi] \equiv \int_0^\infty dE' \int_{4\pi} d\mathbf{\Omega}'\Sigma_s(\mathbf{r};E',\mathbf{\Omega}'\to E,\mathbf{\Omega})\phi(\mathbf{r},E',\mathbf{\Omega}') \quad (11)$$

is the scattering source term.

3. The quantity

$$Q_f[\phi] \equiv \int_0^\infty dE' \int_{4\pi} d\mathbf{\Omega}'[\chi(E'\to E)\nu(\mathbf{r},E')\Sigma_f(\mathbf{r},E')/4\pi]$$
$$\times\phi(\mathbf{r},E',\mathbf{\Omega}') \quad (12)$$

is the fission source term.

Note that both $Q_s[\phi]$ and $Q_f[\phi]$ are linear functionals of the flux ϕ. The eigenvalues α, γ, and δ, which appear in Eqs. (8), (9), and (10), respectively, represent the asymptotic inverse reactor period, the effective collision multiplication factor, and the effective density factor.

Equations (8), (9), and (10) are being solved iteratively. The flux obtained in the $(n-1)$'th iteration, $\phi^{(n-1)}$, is used to calculate the source term for the n'th iteration, $Q[\phi^{(n-1)}]$, and the latest eigenvalue [$\alpha^{(n-1)}$, $\gamma^{(n-1)}$, and $\delta^{(n-1)}$] is being used to calculate the flux in the n'th iteration. Equations (8), (9), and (10) can thus be rewritten as

$$[\alpha^{(n-1)}/v + \mathbf{\Omega}\cdot\nabla + \Sigma_t]\phi_\alpha^{(n)} = Q[\phi_\alpha^{(n-1)}] \ , \quad (8')$$

$$(\mathbf{\Omega}\cdot\nabla + \Sigma_t)\phi_\gamma^{(n)} = \frac{1}{\gamma^{(n-1)}}Q[\phi_\gamma^{(n-1)}] \ , \quad (9')$$

and

$$\left[\mathbf{\Omega}\cdot\nabla + \frac{\Sigma_t}{\delta^{(n-1)}}\right]\phi_\delta^{(n)} = \frac{1}{\delta^{(n-1)}}Q[\phi_\delta^{(n-1)}] \ . \quad (10')$$

The new estimates for the eigenvalues are derived from

$$\alpha^{(n)} = \alpha^{(n-1)} + \frac{S[\phi_\alpha^{(n)}] - S[\phi_\alpha^{(n-1)}]}{W[\phi_\alpha^{(n)}]} \ , \quad (13)$$

$$\gamma^{(n)} = \gamma^{(n-1)}\frac{S[\phi_\gamma^{(n)}]}{S[\phi_\gamma^{(n-1)}]} \ , \quad (14)$$

and

$$\delta^{(n)} = \delta^{(n-1)}\frac{S[\phi_\delta^{(n)}] - T[\phi_\delta^{(n)}]}{S[\phi_\delta^{(n-1)}] - T[\phi_\delta^{(n)}]} \ . \quad (15)$$

The quantities appearing (in abbreviated notation) in the preceding equations are defined as follows:

1. $S[\phi^{(n)}]$ is essentially the integral over phase space of the total source term $Q[\phi^{(n)}]$
2. $W[\phi_\alpha^{(n)}]$ is the integral over phase space of $1/v$ times the flux $\phi_\alpha^{(n)}(\mathbf{r},E,\mathbf{\Omega})$
3. $T[\phi_\delta^{(n)}]$ is the integral over phase space of Σ_t times the flux.

In conclusion, we are grateful to Dr. Hill for his comments that provided an opportunity to stress once again what is rigorous and what is a very good approximation. We are also pleased to be able to introduce the algorithms for the direct evaluation of various eigenvalues. It is a pity that Hill did not make any comment on the closing remarks of Ref. 1.

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Propagation of Knowledge Regarding Conservation During Doppler Broadening

Canfield[1] in 1967 and later Cullen[2] in 1973 demonstrated that the area under the cross-section curve is not conserved and that the only conservation law associated with Doppler broadening is the integral of the reaction rate per incident neutron over all phase space. It was also emphasized by Cullen[2] that "the concept of conservation of area under the curve being conserved has propagated into the literature and become part of the folklore of nuclear engineering." Cullen and Weisbin[3] present an interpretation of the Doppler broadening equations in Eqs. (13) through (19). On pp. 209-211 they explicitly discuss the psi-chi method and its limitations, and Table III on p. 210 contains a concise summary of conservation laws for the exact and various approximations to the Doppler broadening equation.

In this Letter we point out that the books very recently published have not taken note of these facts published a decade ago and they continue to erroneously propagate the concept of conservation of area during Doppler broadening. For instance Waltar and Reynolds[4] state, ". . . infinite dilute group cross section is unaffected by Doppler broadening." Ash[5] makes the same error by stating that "the average cross section over the resonance region stays constant" (independent of temperature). In Ref. 6, a research monograph, it is stated by Rowlands on p. 57 that the total area under the resonance curve is constant when the temperature changes. In the same monograph,[6] James and de Saussure state on p. 136, "It can be shown that Doppler broadening conserves the 'area' under a resonance."

The author of the present Letter sincerely hopes that at

[1]E. CANFIELD, "On the Models for Calculating Effects of Thermal Moderator Motion," UCRL-50323, Lawrence Livermore National Laboratory (1967).

[2]D. E. CULLEN, *Nucl. Sci. Eng.*, **52**, 498 (1973).

[3]D. E. CULLEN and C. R. WEISBIN, *Nucl. Sci. Eng.*, **60**, 199 (1976).

[4]A. E. WALTAR and A. B. REYNOLDS, *Fast Breeder Reactors*, p. 166, Pergamon Press (1981).

[5]M. ASH, *Nuclear Reactor Kinetics*, 2nd Ed., McGraw-Hill Book Company (1979).

[6]J. L. ROWLANDS, "Fission Cross Section Requirements for the Nuclear Energy Programme," p. 57, and G. D. JAMES and G. de SAUSSURE, "Measurements of Fission Cross Sections," p. 89, *Nuclear Fission and Neutron-Induced Fission Cross Sections*, A. MICHAUDON, Ed., Pergamon Press (1981).

least in future editions of these books and in other books in nuclear science and engineering that may be published in the future, such erronous conceptual statements will not be made. It should be clearly pointed out to students in nuclear science and engineering that the area under the curve of a cross section is conserved only under the so called psi-chi approximation. The detailed equations and discussions relating to exact Doppler broadening are not reproduced here in order to save space and are readily available in Ref. 3.

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Responses to "Propagation of Knowledge Regarding Conservation During Doppler Broadening"

I hope that Ganesan[1] is overestimating the dangers of students being misled by statements such as "the total area under the resonance curve is constant when the temperature changes" and that they will ask the questions:

1. What is the variable of integration (energy, velocity, lethargy, etc.)?
2. What is the range of integration?
3. How does this relate to the quantity of relevance in reactor calculations (the integral of flux times cross section)?

The integral that remains constant when temperature changes is

$$\int_0^{\infty} E\sigma(E,T)\,dE \ .$$

I would agree that a student is unlikely to guess that it is the area under the curve of σ plotted against E^2 that remains constant. However, in most cases of practical interest the resonances are sufficiently narrow for the area to remain approximately constant when the variable of integration is energy (or even lethargy).

When the self-shielding and mutual shielding effects are small, the ratio of integrals

$$\int_{E_1}^{E_2} \phi(E)\sigma(E,T)\,dE \Big/ \int_{E_1}^{E_2} \phi(E)\,dE$$

can often be approximated as independent of temperature, but the student should carefully consider whether this is true for the particular range of integration and flux shape, $\phi(E)$, in relation to the widths and positions of the resonances in the energy interval (E_1 to E_2), and also whether $\phi(E)$ is itself a function of T.

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[1]S. GANESAN, *Nucl. Sci. Eng.*, **86**, 118 (1984).

In his Letter to the Editor, Ganesan[1] has supplied a number of references in which the details of Doppler broadening are presented. Therefore, I will only present a few comments here on conservation of "the area under the curve" of resonances and the related assumption that "Doppler broadening smooths cross sections."

First of all let me state that one should not be too hard on authors who state that "the area under the curve" of resonances is conserved. In textbooks and other references that introduce the psi-chi approximation for use in fission reactor core calculations, a natural result of introducing the psi-chi approximation is the observation that, when the psi-chi approximation is used, the "area under the curve" of a resonance is conserved, and under Doppler broadening, cross sections become smoother. If restricted to the energy and temperature ranges where the psi-chi approximation is valid, it is an excellent tool that is both economical and accurate for use in predicting the behavior of resonances under Doppler broadening. Therefore the introduction of psi-chi approximating and its consequences in textbooks is certainly worth doing, since it introduces the reader to a very practical method that is widely used in fission reactor calculations.

However, what is not stressed in textbooks is identification of the range of validity of the psi-chi method and recognition that conservation and smoothing of the cross section are a consequence of the psi-chi approximation and are not properties of the basic Doppler broadening equation. In particular, the failure to explicitly point out that conservation and smoothing of cross section are a result of using the psi-chi approximation has led readers to assume that these are general properties of Doppler broadening, and they have applied these concepts to applications where they are not valid.

The basic Doppler broadening equation conserves and smooths the reaction rate [$N\sigma(N)$], not cross sections. In the higher energy resonance region at fission reactor temperatures, where the reaction shape is dominated by resonance profiles, distinguishing between reaction and cross-section smoothing is of little practical concern. It is in this energy range that the psi-chi approximation is valid and accurate and where, for all practical purposes, cross-section conservation and smoothing occur. However, even here care must be exercised to define the cross section accurately over the entire energy range, particularly for heavy even-even isotopes such as ^{232}Th, ^{238}U, and ^{240}Pu where the resonances are widely spaced; e.g., for ^{238}U, on average, the resonances are some 500 half-widths apart.

In many modern evaluations, the "resonance region" extends to very low energies, well below the energy range in which resonance peaks occur; e.g., in many ENDF/B evaluations, the resonance region extends down to 10^{-5} eV. At low energies, distinguishing between reaction and cross-section conservation can be very important. The low energy limit of Breit-Wigner resonances is not a zero cross section; the capture and fission cross sections become $1/v$ and the elastic constant. Since the capture and fission reaction rate at low energies is constant, the reaction rate is already "smooth," and as such these cross sections are essentially independent of temperature. In contrast, the constant elastic cross section is temperature dependent; this is true of all cross sections, which at zero Kelvin are constant at low energies whether they are defined by a series of resonances or simply in the tabulated form used in many modern evaluations. An initially constant cross section under Doppler broadening will develop a $1/v$ tail

[1]S. GANESAN, *Nucl. Sci. Eng.*, **86**, 118 (1984).