Letters to the Editor

Comments on Ash's paper: "Application of Dynamic Programming to Optimal Shutdown Control"

Ash[1] has recently analyzed the xenon shutdown problem using a minimax criterion of optimality. In our work, it was demonstrated that a) the solutions to the minimum time shutdown problem are either single-pulse trajectories or trajectories that follow the xenon boundary[2], and b) the minimax problem (criterion (i) of Ash) is equivalent to the minimum time problem[3]. Since many of the solutions proposed by Ash are multipulse, we question whether they are optimal. His work develops optimal one-pulse solutions for an unconstrained Xe-I state space but does not present any mathematical reasoning for introducing multiple pulses. Only computational results are shown. It would be helpful if more detail could be developed to show how the multipulse solutions arise.

Figure 4 of Ref. 1 poses an interesting question. It shows an "ideal extremal path if sufficient built-in reactivity ($62.5) is available to override xenon at onset of first optimal control pulse[1]." However, the ultimate coasting or target curve, which is reached at the end of the 9.58 hour controlled shutdown, has a maximum of less than $40. This solution is unrealistic from the standpoint of reactor operation since the maximum value of the xenon concentration should be minimized for the period of time beginning with the shutdown program rather than the period of time beginning at the end of the shutdown program[3,4]. Why trouble to reduce the peak to $40 if there are $62.5 available?

In Fig. 8 of Ref. 1, the 9.58 hour (T=1) program ends on the coasting phase trajectory (or target curve) at the normalized xenon value $X/X(0)$ = 0.5. However, prior to this, the shutdown trajectory crosses the target curve at $X/X(0) = 3$. Therefore, the same coasting phase trajectory (i.e., the same minimax) could have been achieved in *less* time than the duration of the optimal program depicted in this Figure.

In the last paragraph of his article, Ash mentions the advisability of traveling along the boundary "if the flux constraint upper bound is relaxed." This bound M deter- the allowable power range as follows:

$$0 \leq u = \phi/\phi_{\text{equilibrium}} \leq M = \phi_{\max}/\phi_{\text{equilibrium}} , \qquad (1)$$

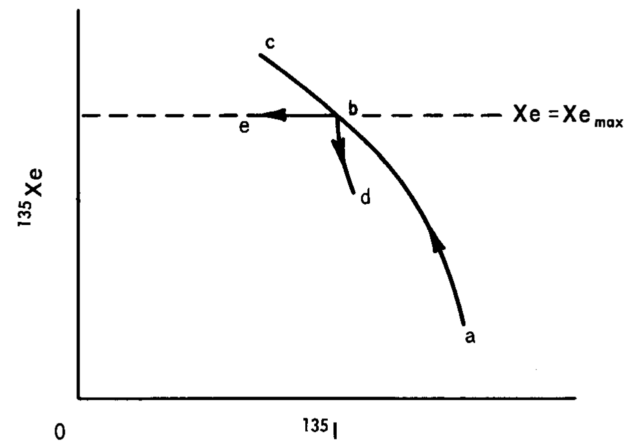

Fig. 1. Alternatives at the boundary in Xe-I space. Point *a* represents equilibrium at $\phi = \phi_{\text{equilibrium}} = \phi_{\max}$. Segment *abc* is at $\phi = 0$. Segment *bd* is a maximum flux pulse ($\phi = \phi_{\max}$).

where M is established by operational procedures for a given reactor. For convenience, M is assumed equal to 1.0 for the numerical results in Refs. 1 through 4. If this is the case, the boundary Xe = $\text{Xe}_{\max}$ can be followed without violating the flux constraint. From the xenon and iodine equations in standard form (e.g., Eqs. 1 and 2 of Ref. 1), the flux ϕ_c necessary to turn the corner and follow the boundary is

$$\phi_c(\tau) = \frac{\lambda_I I(\tau) - \lambda_X X(\tau)}{\sigma_X X(\tau) - \gamma_X \Sigma_f}, \qquad (2)$$

where τ is the time at zero flux to arrive at the boundary. The special case $\tau = 0$ reduces to $\phi_c(0) = \phi_{\text{equilibrium}}$; that is, $u_c(0) = M = 1$. Now for $\tau > 0$, it is clear that $I(\tau)$ will decrease and $X(\tau)$ will increase. Thus for $\tau > 0$, that is for any realistic problem, $u_c(\tau) < 1$. This is intuitively obvious in Fig. 1 of this note. Point *a* corresponds to equilibrium with u=M=1. Curve *abc* is the zero flux curve crossing the xenon boundary at *b*. The full power solution of Ash would follow trajectory *abd* with u=M=1 on segment *bd*. Clearly the optimal trajectory *abe* will not violate the power constraint since it lies between curves *abc* and *abd*. In fact for most examples of interest, the value[2] of u on segment *bc* will be less than 0.5.

Ash also comments in his last paragraph that "u_c can become negative." This will not occur for mathematically optimal shutdown programs[2].

*John J. Roberts**
Harold P. Smith, Jr.

University of California
Berkeley, California
January 24, 1966

*Present address: Reactor Engineering Division, Argonne National Laboratory, Argonne, Illinois 60440

[1] M. ASH, "Application of Dynamic Programming to Optimal Shutdown Control," *Nucl. Sci. Eng.*, **24**, 77-86 (1966).
[2] J. J. ROBERTS and H. P. SMITH, Jr., "Time Optimal Solution to the Reactivity-Xenon Shutdown Problem," *Nucl. Sci. Eng.*, **22**, 470-478 (1965).
[3] J. J. ROBERTS and H. P. SMITH, Jr., "Equivalence of the Time Optimal and Minimax Solutions to the Xenon Shutdown Problem," *Nucl. Sci. Eng.*, **23**, 397-399 (1965).
[4] Z. R. ROSZTOCZY and L. E. WEAVER, "Optimum Reactor Shutdown Program for Minimum Xenon Buildup," *Nucl. Sci. Eng.*, **20**, 318-323 (1964).