Letter to the Editors

On the Determination of the Transfer Function through a "Nonlinear" Measurement

In order to measure the dynamic behavior of a noisy reactor by an oscillation method it is desirable that the amplitude of the power oscillations be comparatively large. The ratio of the first harmonic of the reactor response to the first harmonic of the reactivity is given in this case by the describing function, which depends in a complicated way on the kinetic parameters, especially in higher approximations. For comparison of measurements with the theory it is, however, more convenient to refer to the transfer function of the corresponding linear system (for instance in measurements of the neutron lifetime).

The purpose of this note is to point out that the transfer function of a zero power reactor can be determined by measuring the first two harmonics of the reactor response to a sinusoidal reactivity variation of arbitrary amplitude. The limitation is given by the range of validity of the kinetic equations

$$\begin{aligned} \Lambda \dot{n} &= \rho n - \beta n + \Lambda \sum_i \lambda_i C_i \\ \dot{C}_i &= \frac{\beta_i}{\Lambda} n - \lambda_i C_i \end{aligned} \tag{1}$$

(in the standard notation).

Assume that the reactor is stabilized during the oscillations. Replace all the time dependent quantities in Eq. (1) by their Fourier expansions of the type

$$x(t) = \sum_{m=-\infty}^{\infty} x_m e^{jm\omega t}, \qquad x_m = x_{-m}^{+}, \tag{2}$$

(the plus sign means complex conjugate). Multiplying the equations (1) by $e^{-i\omega t}$ and integrating over the period, one obtains

$$n_1 = T \cdot [\rho_1 n_0 + \sum_{m=1}^{\infty} (\rho_m{}^{+} n_{m+1} + \rho_{m+1} n_m{}^{+})], \tag{3}$$

where

$$T = \left[j\omega\Lambda + \sum_i \frac{j\omega\beta_i}{j\omega + \lambda_i} - \rho_0 \right]^{-1} \tag{4}$$

is the transfer function of a subcritical reactor, the subcriticality being given by

$$\rho_0 = -\frac{2}{n_0} \sum_{m=1}^{\infty} \mathrm{Re}\,(\rho_m{}^{+} n_m) \tag{5}$$

(the subscripts, except i, denote the corresponding coefficients in Fourier series). The quantity ρ_0 was obtained by integrating Eq. (1) over the period.

If the reactivity $\rho(t)$ is given by $\rho(t) = \rho_0 + 2\rho_1 \cos \omega t$, Eqs. (3) and (5) become

$$\frac{n_1}{n_0} = T \cdot \rho_1 \cdot \left[1 + \frac{n_2}{n_0} \right] \tag{3a}$$

$$\rho_0 = -2\rho_1 \,\mathrm{Re} \left(\frac{n_1}{n_0} \right). \tag{5a}$$

This result has been arrived at by Lauber (*1*) and Wasserman (*2*) also. However, they have failed to point out the following important consequences of this result:

A. As may be seen from Eqs. (3a) and (5a), the transfer function

$$Z = \frac{1}{1/T + \rho_0} \tag{6}$$

of a critical reactor can be derived from measurement of n_1/n_0, n_2/n_0, and from known ρ_1.

B. The usual requirement for linearity: $(n(t) - n_0)/n_0 \ll 1$ (i.e. small amplitude of the oscillations) is sufficient, but not necessary for the "linear" equation $n_1/n_0 = T\rho_1$ to be valid. The necessary and sufficient requirement is

$$|n_2/n_0| \ll 1. \tag{7}$$

Generally, if the reactivity variation is odd harmonic, it may be seen from Eq. (3) that the deviation from linearity is given by even harmonics of the reactor response, i.e., by the deviation of the reactor response from an odd harmonic function (an odd harmonic function has an intuitive characteristic: the half wave is repeated but reversed in sign, so that it can easily be recognized).

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REFERENCES

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