In a P_N approximation, one has to take the moments of the balance Eq. (1) in the half-space. The integration is either over all the $\mathbf{\Omega}$ directed into the medium at $\boldsymbol{r}$, or over all the surface S' seen from dS at $\boldsymbol{r}$.

Now, even-order and odd-order spherical harmonics represent a system of orthogonal functions in the half-space. The decision for the even-order spherical harmonics arises from the argument to retain a special balance equation for the current density. This equation follows from Eq. (1) as the zero-moment with $(1,m) = (0,0)$. The cavity boundary conditions in a P_N approximation, therefore, take the form

$$\int_{\mathbf{\Omega}\cdot\boldsymbol{n}<0} Y_1^{*m}\Phi(\boldsymbol{r},\mathbf{\Omega})(\mathbf{\Omega}\cdot\boldsymbol{n})d\Omega = \int_{\mathbf{\Omega}\cdot\boldsymbol{n}<0} Y_1^{*m}\Phi(\boldsymbol{r}',\mathbf{\Omega})(\mathbf{\Omega}\cdot\boldsymbol{n})d\Omega = \int_{S'} Y_1^{*m}\Phi(\boldsymbol{r}',\mathbf{\Omega})\frac{(\mathbf{\Omega}\cdot\boldsymbol{n})(\mathbf{\Omega}\cdot\boldsymbol{n}')}{R^2}\,dS' ,$$

$$1 = 0, 2, 4, \ldots, \quad N-1, \quad N \text{ odd}, \quad -1 \le m \le 1 . \tag{3}$$

For zero exposure, Eq. (3) reduces to the vacuum boundary conditions

$$\int_{\mathbf{\Omega}\cdot\boldsymbol{n}<0} Y_1^{*m}\Phi(\boldsymbol{r},\mathbf{\Omega})(\mathbf{\Omega}\cdot\boldsymbol{n})d\Omega = 0$$

$$1 = 0, 2, 4, \ldots, \quad N-1, \quad N \text{ odd}, \quad -1 \le m \le 1 . \tag{4}$$

In contrast to the generalized Marshak-type boundary conditions cited by Davison[3], Eqs. (3) and (4) represent just the right number of cavity and vacuum boundary conditions for more-dimensional problems.

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[3] B. DAVISON, *Neutron Transport Theory*, Clarendon Press, Oxford, N. Y. (1957).

Fine Group Cross Sections of Resonance Absorbers

Previously[1] a definition of multigroup resonance cross section was given as

$$\Sigma_i = \Sigma_i^* C_i ,$$

where

$$\Sigma_i^* = (1-p_i)\bar{\xi}\Sigma_p/p_i\delta u$$

and

$$C_i = 1 + (\Sigma_g - \Sigma_i^*)\delta u/2\bar{\xi}\Sigma_p . \tag{1}$$

Here, i refers to the i'th resonance of a group g of width δu lethargy units which contains n resonances

$\bar{\xi}$ = average lethargy gain on collision

Σ_p = potential scattering cross section

$\Sigma_g = \sum_{i=1}^{n} \Sigma_i^*$

p_i = resonance escape probability.

[1] A. KEANE and J. P. POLLARD, *Nucl. Sci. Eng.*, 25, 4, 439 (1966).

For a fine group ($\delta u < 2\bar{\xi}$ approximately), the previous derivation needs to be extended to include the possibility of neutrons skipping the group. Consider a group below source energies for a source emitting $1n/(\text{cm}^3\ \text{sec})$ and above thermal energies so that slowing down theory applies. Neutron balance for an infinite homogeneous system gives the equation

$$\sum_{i=1}^{n} \Sigma_i\phi_g + \Sigma_r\phi_g = f , \tag{2}$$

and we will assume that higher groups do not contain absorption. In Eq. (2), ϕ_g is the group flux, Σ_r is the group removal cross section, and f is the probability of a neutron entering the group given by

$$f = \Sigma_r\delta u/\bar{\xi}\Sigma_p = \sum_l \Sigma_{pl}\xi_l f_l/\bar{\xi}\Sigma_p . \tag{3}$$

Here, the summation on l is taken over all nuclides with potential scattering cross section Σ_{pl} and average lethargy gain on collision ξ_l and

$$\left.\begin{aligned} f_l &= [1 - \exp(-\delta u) - \alpha_l\delta u]/\xi_l(1-\alpha_l) , && \delta u < \ln^1/\alpha_l\ (\cong 2\xi_l) , \\ &= 1 && \delta u \ge \ln^1/\alpha_l , \end{aligned}\right\} \tag{4}$$

where α_l is the maximum fractional energy loss on collision with nuclide l.

To obtain the correct reaction rates we must have

$$\Sigma_i\phi_g = \prod_{k=0}^{i-1} p_k(1-p_i), \qquad \text{with } p_0 = 1 , \tag{5}$$

provided we assume the flux to return to asymptotic form between resonances. From Eqs. (2), (3), and (5) we have

$$\phi_g = \left(\prod_{k=1}^{n} p_k + f - 1\right)\delta u/f\bar{\xi}\Sigma_p , \tag{6}$$

and from Eqs. (5) and (6)

$$\Sigma_i = \Sigma_i^* C_i ,$$

where Σ_i^* is given by Eq. (1), but now

$$C_i = \prod_{k=1}^{i} p_k \Big/ \left[1 - \left(1 - \prod_{k=1}^{n} p_k\right)\Big/ f\right] , \tag{7}$$

which must be simplified.

We require an approximation to C_i that preserves neutron balance given by Eq. (2) but which does not distinguish between the actual ordering or resonances in the group. We assume that

$$C_i \simeq 1 + a\left(\sum_{k=1}^{n} \epsilon_k - \epsilon_i\right) + b\epsilon_i , \tag{8}$$

where $\epsilon_i = 1 - p_i$. Equation (2) gives

$$\sum_{i=1}^{n} \epsilon_i \left[1 + a\left(\sum_{k=1}^{n} \epsilon_k - \epsilon_i\right) + b\epsilon_i\right] \Big/ f(1-\epsilon_i) \simeq \left\{1 - \left[1 - \prod_{k=1}^{n}(1-\epsilon_k)\right]\Big/ f\right\}^{-1} - 1 , \tag{9}$$

and we require this to be accurate to terms $\epsilon_i\epsilon_k$. When only one resonance is in the group, we require

$$b = \frac{1}{f} - 1 .$$

Using this choice in Eq. (8) then gives

$$a = \frac{1}{f} - \frac{1}{2} \quad .$$

Equation (8) thus becomes

$$C_i \simeq 1 + \left[\left(\frac{2}{f} - 1\right) \Sigma_g - \Sigma_i^*\right] \delta u \Big/ 2\bar{\xi}\Sigma_p \quad . \qquad (10)$$

For fine groups, Eq. (10) should replace the corresponding Eq. (1). As the width of the group is increased, both equations become identical.

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