TABLE I

The H-Function and its Analytic Approximation

c	0	0.2	0.4	0.6	0.8	1.0
μ	$H_{22}^{(1)} = \left[1 - \frac{\alpha c}{2}\mu - (1-\alpha\mu)\frac{c}{2}\mu \ln\frac{1+\mu}{\mu}\right]^{-1}$					
0	1	1	1	1	1	1
0.2	1	1.03871	1.0848	1.1427	1.2233	1.444
0.4	1	1.05523	1.1241	1.2155	1.3540	1.819
0.6	1	1.06564	1.1498	1.2657	1.4511	2.179
0.8	1	1.07297	1.1683	1.3031	1.5276	2.534
1.0	1	1.07844	1.1823	1.3323	1.5899	2.885
H-function						
0	1	1	1	1	1	1
0.2	1	1.03892	1.08577	1.14517	1.2286	1.4503
0.4	1	1.05546	1.12516	1.21861	1.3611	1.8293
0.6	1	1.06587	1.15087	1.26893	1.4590	2.1941
0.8	1	1.07319	1.16935	1.30631	1.5358	2.5527
1.0	1	1.07864	1.18337	1.33541	1.5982	2.9078

TABLE II

First Moment of the H-Function; $\alpha_1^{[*]} = \int_0^1 H(\mu)\mu d\mu$;

$\alpha_1 \simeq \bar{\alpha}_1 = \int_0^1 \frac{1}{2}(H_{21}^{(1)} + H_{11}^{(1)})\mu d\mu$

c	0	0.1	0.2	0.3	0.4	0.5
α_1	0.5	0.515609	0.533154	0.553123	0.576210	0.603495
$\bar{\alpha}_1$	0.5	0.51553	0.53316	0.55314	0.57624	0.60352
c		0.6	0.7	0.8	0.9	1.0
α_1		0.636636	0.678674	0.735808	0.825318	1.154701
$\bar{\alpha}_1$		0.63665	0.67862	0.7355	0.8242	1.145

[*] Deduced from Tables XI and XXXIII of Chandrasekhar[3] which are accurate to within 0.005%.

TABLE III

(b) $1 - R(\mu_0 = 1)$

c	0	0.2	0.4	0.6	0.8	1.0
Rafalski	1	0.96306	0.9076	0.8222	0.6716	0
Pomraning	1	0.96391	0.9137	0.8398	0.7078	0
$H \approx H_{22}^{(1)}$	1	0.96459	0.9158	0.8426	0.7110	0
Chandrasekhar[a]	1	0.96476	0.9166	0.8446	0.7147	0

[a]Deduced from Tables XI and XXXIII of Chandrasekhar[3] which are accurate to within 0.005%.

$H \approx H_{11}^{(1)}$ or $H_{21}^{(1)}$. For a simple expression for the first moment of the H-function, we found it best to approximate H by $H \approx \frac{1}{2}[H_{11}^{(1)} + H_{21}^{(1)}]$. This choice leads to

$$\alpha_1 \simeq \bar{\alpha}_1 = \frac{1}{2} + \left(\frac{\alpha}{3} + \frac{c}{6}\right)\left(\ln 2 - \frac{1}{4}\right) + \frac{c\alpha}{12} - \frac{c\alpha^2}{10}\left(\ln 2 - \frac{11}{12}\right). \quad (15)$$

This result is compared in Table II with the first moments of the H-function α_1 given by Table XXXIII of Chandrasekhar[3].

Equations (9) through (15) when applied to Eqs. (1) and (2) constitute the present approximations for the albedos. The approximations of Eqs. (1), (14) and (2), (15) for albedo defects $1-R(1)$ and $1-\bar{A}_{is}$ are compared in Tables III and IV with the corresponding results of Rafalski and Pomraning, and with values computed from Tables XI and XXXIII of Chandrasekhar[3] as a reference.

This work was done under the supervision of G. Birkhoff. Further work on the above method of approximation has been done and an extension is in progress.

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Cavity and Vacuum Boundary Conditions for More-dimensional P_N Approximations

In the June 1966 issue of *Nuclear Science and Engineering*, Davis[1] has derived vacuum boundary conditions for the neutron flux and its adjoint in a P_N approximation (N even and odd) by variational methods.

Equivalent boundary conditions have been obtained by neutron balance considerations[2]. A natural approach is to start with cavity boundary conditions for exposed surfaces. The number of neutrons that enter a surface element $d\boldsymbol{S} = \boldsymbol{n}\cdot dS$ per s in a solid angle $d\Omega$ around $\boldsymbol{\Omega}$ is determined by neutrons that leave the surface element $d\boldsymbol{S}' = \boldsymbol{n}'\cdot d\boldsymbol{S}'$ in the solid angle $-(\boldsymbol{\Omega}\cdot\boldsymbol{n})dS/R^2$ around $\boldsymbol{\Omega}$

$$-\Phi(\boldsymbol{r},\boldsymbol{\Omega})(\boldsymbol{\Omega}\cdot\boldsymbol{n})d\Omega\, dS = \Phi(\boldsymbol{r}',\boldsymbol{\Omega})(\boldsymbol{\Omega}\cdot\boldsymbol{n}')\left[-\frac{(\boldsymbol{\Omega}\cdot\boldsymbol{n})}{R^2}\right] dS\cdot dS' \quad . \quad (1)$$

Hence, $d\Omega$ is related to dS' by

$$d\Omega = (\boldsymbol{\Omega}\cdot\boldsymbol{n}')\cdot dS'/R^2 \quad . \quad (2)$$

[1]J. A. DAVIS, "Variational Vaccum Boundary Conditions for a P_N-Approximation," *Nucl. Sci. Eng.*, 25, 2, 189 (1966).

[2]D. EMENDÖRFER, "Randbedingungen für den Neutronenfluz im endlichen Zylinder nach der P_N-Approximation der Transportgleichung," *Nukleonik*, 5, 74 (1963).

TABLE IV

$1 - \bar{A}_{is} = 2\alpha_1\sqrt{1-c}$

c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Pomraning	1	0.980	0.957	0.929	0.896	0.856	0.808	0.746	0.660	0.523	0
$\alpha_1 \approx \bar{\alpha}_1$	1	0.97814	0.95375	0.92558	0.89270	0.85351	0.80494	0.7434	0.6579	0.5213	0
Chandrasekhar[a]	1	0.97830	0.95374	0.92555	0.89266	0.85347	0.80529	0.74345	0.65813	0.52198	0

[a]Deduced from Tables XI and XXXIII of Chandrasekhar[3] which are accurate to within 0.005%.