P alone (which of course is no operator) refers to future integration. Never did we make the statement, of which we are accused, that the Poincaré-Bertrand formula is ambiguous, but only that one of the integrals involved requires an interpretation, and that with a different interpretation the shape of the formula changes. Kaper too quotes the usual interpretation, equivalent to our Eq. (3B), after his Eq. (8).

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Further Comments on the Use of Generalized Functions in Neutron Transport Theory

We are indeed indebted to Mr. Kaper[1] for his clarification of the arguments introduced by Jacobs and McInerney[2] and Kuščer and McCormick[3] regarding the angular Green's function of neutron transport theory. However, we believe it is unfortunate that there is a continued use of the obscure notation of the Poincaré-Bertrand formula[4] which we believe has led to all the difficulties.

In these further comments we shall add two points to the discussion: 1) We shall derive expression (7-3) of reference 1 without recourse to the Poincaré-Bertrand formula and in so-doing obtain an alternative (correct in the sense of distributions) to equation (3A) of reference 3. 2) For the sake of those who do not desire to become intimatelv acquainted with the involvements of distribution theory, we shall attempt to discuss the practical implication of these considerations.

Referring to reference 5 for mathematical background and notation we state two easily proven lemmas:

Lemma 1: $(x+i0)^{-1} = x^{-1} - i\pi\,\delta(x)$,

where x^{-1} is the canonical regularization (i.e. the Cauchy principal value).

Lemma 2: $\mu(\nu'-\mu+i0)^{-1}\ (\nu-\mu+i0)^{-1}$

$$= (\nu-\nu'+i0)^{-1}\ (\nu'(\nu'-\mu+i0)^{-1}-\nu(\nu-\mu+i0)^{-1}),$$

where all terms are distributions with respect to the three variables μ, ν' and ν. We use these lemmas to prove a theorem of central importance in the consideration of generalized functions relevant to transport theory.

Theorem: $\mu(\nu'-\mu)^{-1}(\nu-\mu)^{-1}$

$$= (\nu-\nu')^{-1}(\nu'(\nu'-\mu)^{-1}-\nu(\nu-\mu)^{-1}) + \pi^2\mu\delta(\nu'-\mu)\delta(\nu-\mu)$$

With change of variable notation, this result should be compared with equation (3A) of reference 3. Here, specifications on integration order are meaningless. A particular example of the use of this theorem is its application to the unit test function on (-1,+1) with respect to the variable μ. The result is

$$\int_{-1}^{+1} d\mu(\mu(\nu'-\mu)^{-1}(\nu-\mu)^{-1}) = (\nu-\nu')^{-1}\left(\nu'\ln\left[\frac{\nu'+1}{\nu'-1}\right]-\nu\ln\left[\frac{\nu+1}{\nu-1}\right]\right) + \pi^2\nu\delta(\nu-\nu')$$

when ν, ν' are in (-1,+1). This is in agreement with equation (7-3). However, it is obtained without the rather confusing arguments related between equations (8) and (7-3) of reference 1.

With regard to the practical implication of these considerations, we believe that it is important to state that crucial effects of clear notation (in the use of generalized functions in transport theory) occur only in problems where there is a source singular in the angle variable (i.e. the angular Green's function). Even in the case of such problems, a consistent use of the closure condition yields answers which can be correctly interpreted[2,3]. However, as Kaper points out, the notation of these answers may not be that of formal distribution theory (viz., that in reference 5).

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