NUCLEAR SCIENCE AND ENGINEERING: 4, 255–259 (1958)

Letters to the Editors

Flux Disadvantage Factor in Plane Slab Lattices[1]

In a recent paper (*1*), Bengston presented a calculation of the neutron self-shielding in lattices. His calculation assumed a periodic plane lattice of nonscattering absorber, thickness[2] $2a$, and nonabsorbing moderator, thickness $2(b\text{-}a)$. This note points out that under these assumptions (indeed, with only the assumption of small absorption in the moderator), a much simpler calculation is possible.

We assume that diffusion theory is valid in the moderator and that the inverse diffusion length and diffusion coefficient measured in moderator mean free paths are respectively K and D. The flux in the moderator with a constant isotropic source of neutrons is

$$\phi(x) = \frac{S}{D\mathrm{K}^2}[1 - A \cosh \mathrm{K}(b - x)] \tag{1}$$

in which we have imposed the boundary condition of flux symmetry around the plane $x = b$ (the center of the moderator region). The constant A is still to be determined. Using this form for the flux gives us the following consistent P_1 approximation for the angular distribution in the moderator:

$$\psi(x, \mu) = \frac{1}{2}\left(\phi - 3\mu D \frac{d\phi}{dx}\right). \tag{2}$$

The current out of the fuel slab can be written in two ways:

$$\begin{aligned} J_{\text{out}} &= \frac{1}{2}\int_0^1 \mu \left\{\phi(a) - 3\mu D \left.\frac{d\phi}{dx}\right|_a\right\} d\mu \\ &= \frac{1}{2}\int_0^1 \mu \left\{\phi(a) + 3\mu D \left.\frac{d\phi}{dx}\right|_a\right\} e^{-2a/\mu}\, d\mu \end{aligned} \tag{3}$$

from which we find

$$\frac{\phi(a)}{2D\left.\dfrac{d\phi}{dx}\right|_a} = \frac{1 - A\cosh \mathrm{K}(b - a)}{2D\mathrm{K}A \sinh \mathrm{K}(b - a)} = \frac{1 + 3E_4(2a)}{1 - 2E_3(2a)}$$

or

$$\frac{1}{A} = \cosh \mathrm{K}(b - a) + 2D\mathrm{K}\left(\frac{1 + 3E_4}{1 - 2E_3}\right)\sinh \mathrm{K}(b - a) \tag{4}$$

where $E_n = E_n(2a)$ = exponential integral function.

The average moderator flux is

$$\phi_m = \frac{S}{\mathrm{K}^2 D}\left[1 - A\frac{\sinh \mathrm{K}(b - a)}{\mathrm{K}(b - a)}\right]$$

---

[1] Work performed under Contract AT-11-1-GEN 8 with the U.S. Atomic Energy Commission.

[2] Distances are measured in terms of mean free paths in each region.

TABLE I

Method	ϕ_f/ϕ_m
$P_1{}^2$	0.857
$P_2{}^2$	0.814
$P_3{}^2$	0.791
Bengston	0.79 ± 0.02
Eq. (5)	0.802

and the average fuel flux, (evaluated most easily from the condition of conservation of neutrons), is

$$\phi_f = \frac{SA \sinh \mathrm{K}(b - a)}{\mathrm{K}a}.$$

This gives, for the disadvantage factor,

$$\begin{aligned} \phi_m/\phi_f &= a(b - a)G_m + \frac{2a(1 + 3E_4)}{1 - 2E_3} \\ G_m &= \frac{\mathrm{K}(b - a) \coth \mathrm{K}(b - a) - 1}{D\mathrm{K}^2(b - a)^2} \end{aligned} \tag{5}$$

for the limiting case of a nonabsorbing moderator ($\mathrm{K} = 0$)

$$G_m = 1, \qquad D = \tfrac{1}{3}.$$

To see the accuracy of this expression we may use the test problem quoted by Bengston (*1*); $2a = 0.295$, $2(b - a) = 0.1504$, $\mathrm{K} = 0$. The results are given in Table I. The double P_n calculations (*2*) are Bengston's comparison results (*3*). Thus, the approximation developed here is approximately as good as either Bengston's method or Yvon's double-P_3 method.

The formulation of the problem by the present method involves little more work than that required for an elementary diffusion theory calculation while yielding an accuracy intermediate between the $P_2{}^2$ and $P_3{}^2$ methods. It is also an easy calculation to compute the flux distribution in the fuel plate in addition to the flux distribution [from Eq. (1) and Eq. (4)] in the moderator. It is the primary purpose of this note to point out that this method of mixed transport-consistent-P_1 may find useful application in various reactor problems.

REFERENCES

1. J. Bengston, *Nuclear Sci. and Eng.* **3,** 71 (1958).
2. V. Kourganoff, "Basic Methods in Transfer Problems," footnote, p. 101. Oxford U. P., London and New York, 1952.
3. J. Bengston, CF-56-3-170 (1956) (internal ORNL memorandum).

E. Richard Cohen

Atomics International
Canoga Park, California
Received March 31, 1958.